Annals of the University of Petroşani, Mechanical Engineering, 8 (2006), 111-138 111

THE INTRINSIC IMPORTANCE OF THE TRANSMISSION ANGLE IN THE APPROXIMATE SYNTHESIS OF A LINKAGE MECHANISMS

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Abstract: In this paper, we present approximate synthesis procedures for some function generating simple mechanism involving the conditions on the transmission angle.

Keywords: approximate synthesis, transmission angle, simple mechanisms

1. INTRODUCTION

The transmission angle of a simple four-bar linkage is defined as the angle between the direction of motion t_a of the point on the driven member (crank pin B) at which the driving force is applied and the direction of the relative motion t_r of the driven point (crank pin B) with respect to the driving point (crank pin A). In fig. 1, crank pin A of the input crank A_0A drives crank B of the output crank B_0B through the connection rod AB. The direction of motion of crank pin B, shown as t_a, is perpendicular to crank B_0B, and the direction of relative motion of crank pin B with respect to crank pin A, shown as t_r, is perpendicular to connecting rod AB. The angle between lines t_a and t_r is the transmission angle γ. This angle is equal to the angle between the connecting rod and the output

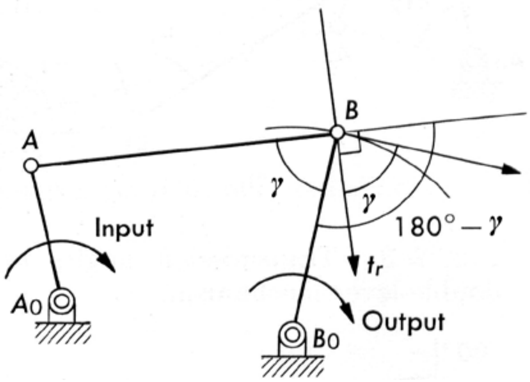

Fig. 1. Definition of transmission angle

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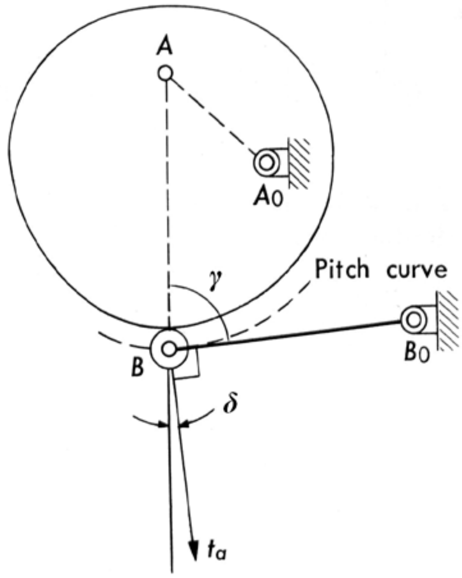

Fig. 2. Relationship between transmission angle γ and pressure angle δ

crank. It may also be defined as $180^0 – \gamma$. In this paper, for the sake of convenience, the transmission angle γ, or $180^0 – \gamma$ (which ever is acute), will be used.

The transmission angle in linkage design is of the same relative importance as the pressure angle in cam design. In fact, the transmission angle is the complement of the pressure angle. The pressure angle of a cam is the angle between the normal of the pitch curve and the direction of motion of the follower. Fig. 2 illustrates the equivalent four-bar linkage A_0ABB_0 to the cam and follower at that particular position. The imaginary crank pin A is the center of curvature of the pitch curve, and the imaginary connecting rod AB is the radius of curvature. The pressure angle δ between the cam and follower is the complement of the transmission angle γ, as shown in the figure.

The importance of the transmission angle is indicated by the following two statements:

a. The force transmission from the connecting rod to the output crank is most effective when the transmission angle is 90^0. Of course, it is most desirable to have this angle deviate from 90^0 as little as possible throughout the range of operation. (In some machinery, a self-locking action is purposely designed, using a very small or zero transmission angles).

b. In some precision machinery, when the transmission angle is too small, the accuracy of the output motion becomes very sensitive to the manufacturing tolerances of the link lengths and clearances between joints. Also, a poor transmission angle can cause objectionable noise and jerk at high speed.

The minimum permissible transmission angle of a linkage mechanism depends on many factors, such as the magnitude of the transmitted forces, the manufacturing tolerances, and the friction between the joints, speed etc. A minimum transmission angle of 40^0 is suggested as a good practical limit by some designers. In some high-speed machinery, a minimum value of 45^0 is necessary. This is a “rule of thumb”. The designer needs to observe the motion and load of mechanisms and develop an intuitive feel to help him decide the minimum transmission angle for a particular design.

So, the transmission angle of a four bar mechanism is formed between the direction of joint C absolute velocity and its relative velocity in relation to joint B, fig.3.

In order to have a mechanism with good transmission characteristics of the force from the connecting rod BC to the effector element CD, the value of the transmission angle γ should range between the limits (γ_{ad} being allowable transmission angle):

$$\gamma_{min} \geq \gamma_{ad} \text{ and } \gamma_{max} \leq 180^0 - \gamma_{ad} \tag{1}$$

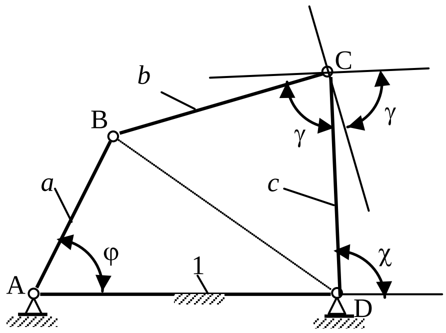

Fig. 3. Transmission angle γ, input angle α and output angle χ

The transmission angle γ can be expressed function of the lengths of the mechanism sides and of the independent variable φ, applying the cosine theorem in the triangles ABD and BCD:

$$BD^2 = b^2 + c^2 - 2\, b\, c\cos\gamma = 1 + a^2 - 2\, a\cos\varphi \tag{2}$$

or

$$\cos\gamma = \frac{b^2 + c^2 - a^2 - 1}{2bc} + \frac{a}{bc}\cos\varphi \tag{3}$$

As the mechanism allows the crank, the angle φ can take values comprised between 0^0 and 360^0. It can be noticed from relation (3) that the values of angle φ leading to extreme values for the transmission angle are 180^0, respectively 0^0 and 360^0, thus:

- for $\varphi = 180^0$, we get γ_{max}:

$$\cos\gamma_{max} = \frac{b^2 + c^2 - (1+a)^2}{2bc} \tag{4}$$

- for $\varphi = 0^0$, 360^0, we get γ_{min}:

$$\cos\gamma_{min} = \frac{b^2 + c^2 - (1-a)^2}{2bc} \tag{5}$$

Consequently, for the transmission angle in a four bar mechanism to take values within the limits expressed by relation (1) and for the driving element AB to be a crank, the following two inequalities (resulting from the expressions (4) and (5) taking into account relations (1)) have to be met:

$$2bc\cos\gamma_{ad} \geq b^2 + c^2 - (1-a)^2 \tag{6}$$

and

$$2bc\cos\gamma_{ad} \geq (1+a)^2 - b^2 - c^2 \tag{7}$$

2. THE ALGORITHM PRESENTATION FOR THE FOUR-BAR MECHANISM WITH DOUBLE CRANK

It is known that the four-bar mechanism with double crank is a complex four bar mechanism, where the shortest element is the base (Grashoff conditions). Further we present an approximate synthesis algorithm for a function generating four bar mechanism with two cranks, imposing conditions on the transmission angle between the connecting rod and the driven crank.

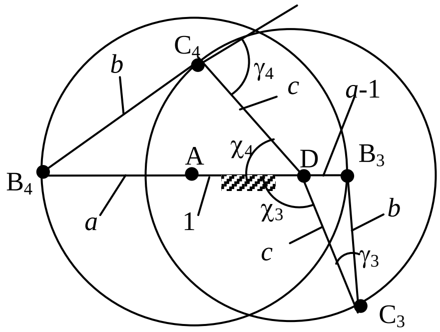

Fig. 4. The mechanism limit positions

Let the four bar mechanism with two cranks in fig. 4, in the limit positions AB_3C_3D and AB_4C_4D, corresponding to the angles of the driving crank $\varphi = 0^0$, respectively $\varphi = 180^0$.

The transmission angles between the connecting rod and the driven crank have been written down with γ_3 and γ_4 and χ_3 and χ_4 marking the positioning angles of the driven crank in relation to the direction of the support AD, while the relative lengths of the mechanism sides have been noted down with a = AB, b = BC, c = CD and 1=AD. Considering the mechanism in the limit position AB_3C_3D, it is easy to find out that:

$$DB_3 = AB_3 - AD = a - 1 \tag{8}$$

and that B_3 is identical with the rotation instantaneous center in the relative motion of the elements AB and CD.

Taking into account the fact that B_3 coincides with the instantaneous rotation center $I_{13} \equiv I_{31}$, the relative velocity of the point B_3 is equal to 0, hence the equality relation of absolute velocities can be written:

$$a\mathrm{d}\varphi = (a\text{-}1)\mathrm{d}\chi \tag{9}$$

Writing down with i = dχ/dφ the transmission ratio between the two cranks, the relation (9) can be rewritten this way:

$$a = \frac{i_3}{i_3 - 1} \tag{10}$$

Applying the sine theorem in the triangle B_3C_3D the following relations can be written and the relative lengths of the sides BC and CD can be determined from them:

$$BC = b = \frac{\sin\chi_3}{(i_3 - 1)\sin\gamma_3} \tag{11}$$

$$CD = c = \frac{\sin(\chi_3 - \gamma_3)}{(i_3 - 1)\sin\gamma_3} \tag{12}$$

Considering the limit position AB_4C_4D, where the point B_4 coincides with the same instantaneous rotation center $I_{13} \equiv I_{31}$, but in another position, the equality relation (9) can be written:

$$a\mathrm{d}\varphi = (a+1)\mathrm{d}\chi \tag{13}$$

Starting from those mentioned above, relation (13) becomes:

$$a = \frac{i_4}{1 - i_4} \tag{14}$$

Applying the sine theorem in the triangle B_4C_4D and taking into account relation (14), the relative lengths b and c are obtained:

$$b = \frac{\sin\chi_4}{(1 - i_4)\sin\gamma_4} \tag{15}$$

$$c = \frac{\sin(\gamma_4 - \chi_4)}{(1 - i_4)\sin\gamma_4} \tag{16}$$

Based on the relations (10), (11), (12) and (14), (15), (16) the following equalities can be written:

$$\frac{i_3}{i_3 - 1} = \frac{i_4}{1 - i_4} \tag{17}$$

$$\frac{\sin\chi_3}{(i_3 - 1)\sin\gamma_3} = \frac{\sin\chi_4}{(1 - i_4)\sin\gamma_4} \tag{18}$$

$$\frac{\sin(\chi_3 - \gamma_3)}{(i_3 - 1)\sin\gamma_3} = \frac{\sin(\gamma_4 - \chi_4)}{(1 - i_4)\sin\gamma_4} \tag{19}$$

The equalities (17), (18) and (19) make up a system of three equations with four unknowns: i_3, i_4, χ_3 şi χ_4, for the solution of which one of the unknowns has to be chosen arbitrarily. Let, for example, χ_4 are chosen arbitrarily. By dividing equality (19) to equality (18), we get the following equality:

$$\frac{\sin(\chi_3 - \gamma_3)}{\sin\gamma_3} = \frac{\sin(\gamma_4 - \chi_4)}{\sin\gamma_4} \tag{20}$$

Having been developed and ordered, relation (20) becomes:

$$ctg\,\chi_3 = ctg\,\gamma_3 - \frac{\sin(\gamma_4 - \chi_4)}{\sin\gamma_3 \sin\chi_4} \tag{21}$$

As it has been shown above, depending on the values of the transmission angles in the two limit positions, γ_3 and γ_4, and by choosing the value of the position angle χ_4, the corresponding value of the angle χ_3 can be calculated with relation (21).

The transmission ratio i_4 results from the relation (17):

$$i_4 = \frac{i_3}{2i_3 - 1} \tag{22}$$

The transmission ratio i_3 is obtained from the relation (18), where we take into account relation (22):

$$i_3 = \frac{1}{2}\left(\frac{\sin\chi_3 \sin\gamma_4}{\sin\chi_4 \sin\gamma_3} + 1\right) \tag{23}$$

At this level, we know the values χ_3 (from relation (21)) and i_3 (from relation (23)) which, by being introduced into relations (10), (11) and (12), allow us to determine the unknown parameters *a*, *b* and *c* of the four bar mechanism with double crank.

It should be mentioned that by choosing the value of angle χ_4 arbitrarily, the resulting mechanism may happen not to be of the two crank types. Obviously, such solutions are left aside. It can be stated, from those mentioned above, that the result depends to a large extent on the arbitrarily chosen value for the position angle χ_4.

Consequently, in order to obtain acceptable solutions, in [1] it is recommendable that the value of the angle χ_4 should be chosen with the relation $\chi_4 = k\,\gamma_{ad}$, with $0 < k < 1$, where γ_{ad} is the allowable transmission angle.

The solutions of the obtained mechanism will be checked to be of the two crank types with the relations:

- $a > 1$, $b > 1$, $c > 1$ and
- 1 + the length of the longest element < the length sum of the other two elements.

For the particular case $\gamma_3 = \gamma_4 = \gamma_{ad}$ (the case when we have the most advantageous transmission conditions) relations (21) and (23) become:

$$\text{ctg}\,\chi_3 = 2\,\text{ctg}\,\gamma_{ad} - \text{ctg}\,\chi_4 \tag{24}$$

$$i_3 = \frac{1}{2}\left(\frac{\sin\chi_3}{\sin\chi_4} + 1\right) \tag{25}$$

2.1. Example of calculation

Determine the relative lengths of the sides of a four bar mechanism with two cranks, having the most advantageous transmission conditions for a minimum allowable angle $\gamma_{ad}=50^0$.

Solution:

1) The angle $\chi_4 = 0{,}8\gamma_{ad} = 40^0$ is chosen.

2) γ_3 is determined with relation (24): $\gamma_3=115{,}94^0$.

3) i_3 is determined with relation (25): $i_3 = 1{,}2$.

4) The parameters *a*, *b* şi *c* are determined with relations (14), (15) şi (16): *a* = 6,01; *b* = 5,88; *c* = 5,97. It is found that AB = *a* is the longest element.

5) Checking: the mechanism is a complex double cranked four bar one, because *a*, *b*, *c* > 1 and *a* + 1 < *b* + *c*.

6) The positional analysis of the obtained mechanism is undertaken.

To this end, we assume the mechanism to be in a certain position, in relation to a reference system with the origin in the joint A and the axis Ox overlapping the direction AD, fig. 5. Our aim is to determine the positional relation $\chi_4 = \chi_4(\varphi)$, where $0 \le \varphi \le 360^0$.

Starting from the notations in fig. 5, the positional analysis algorithm is carried out according to the following steps:

6.1) The coordinates of the mobile joint B are determined:

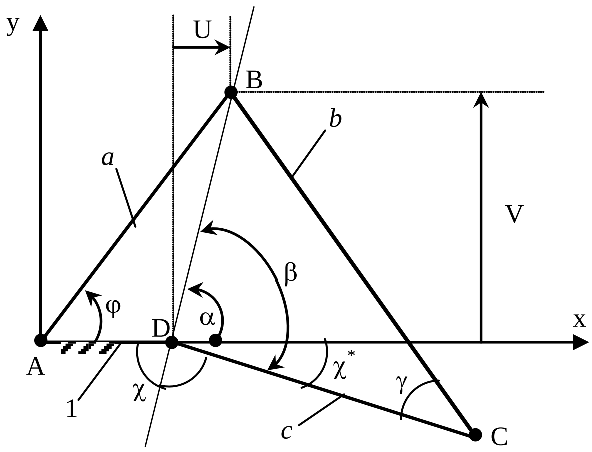

Fig. 5. The positional analysis

$$x_B = a\cos\varphi;\ y_B = a\sin\varphi$$

6.2) The coordinates of the fixed joint D are determined:

$$x_D = 1;\ y_D = 0$$

6.3) The values *U* and *V* are calculated:

$$U = x_B - x_D = a\cos\varphi - 1;\ V = y_B - y_D = a\sin\varphi$$

6.4) The distance BD = *d* is calculated:

$$d = \sqrt{U^2 + V^2} = \sqrt{a^2 + 1 - 2a\cos\varphi}$$

6.5) The slope angle α of the direction BD is calculated:

$$\alpha = arctg\frac{V}{U}$$

6.6) The angle β is calculated applying the cosine theorem in the triangle DBC:

$$\beta = \arccos\frac{c^2 + d^2 - b^2}{2dc}$$

6.7) The angle χ^* and then χ are calculated, considering the α oriented angle:

$$\chi^* = \beta - \alpha;\ \chi = 180^0 + \alpha - \beta$$

6.8) The angle γ in the triangle DBC is calculated and then it is compared to γ_{ad} (the result should be $\gamma \geq \gamma_{ad}$):

$$\gamma = \arccos\frac{b^2 + c^2 - d^2}{2bc}$$

6.9) The diagrams $\chi = \chi(\varphi)$ are plotted, for $0 \leq \varphi \leq 360^0$.

The above shown procedure will be repeated for several values of the angles γ_{ad} and χ_4 or χ_3 for various values given to coefficient k in the interval $0 < k < 1$. Suitable solutions will be chosen.

2.2 Conclusions

The solutions, obtained on the basis of the algorithm put forward, largely depend on the arbitrarily chosen solutions for the angles γ_{ad} and χ_4 or χ_3.

The positional analysis of the four bar mechanism with double crank (parallelogram mechanism are excluded) shows that in a kinematic cycle there are intervals where the value of the rotation angle of the driven crank is higher than the value of the rotation angle of the driving crank. This can be used successfully in practical applications.

3. THE ALGORITHM PRESENTATION FOR THE CRANK-ROCKER FOUR-BAR MECHANISM

It is easy to notice that with a crank-rocker mechanism, fig. 6, the transmission angle has the maximum value when the crank is in extension of the support (position AB_4C_4D) and the minimum value when it takes the position AB_3C_3D.

Taking into consideration the possible critical position of the crank-rocker mechanism, fig. 7, we can notice the position AB_3C_3D, where the transmission angle γ has the minimum value. In this position angle γ_3 must be equal to γ_{ad}.

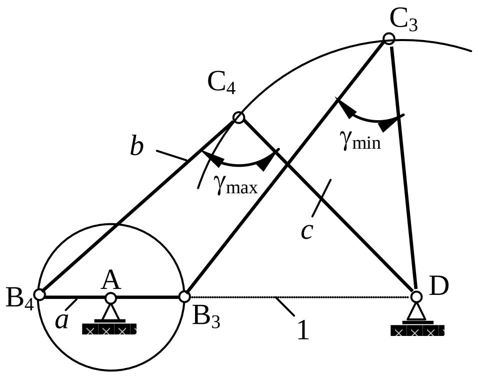

Fig. 6. Four bar crank-rocker mechanism

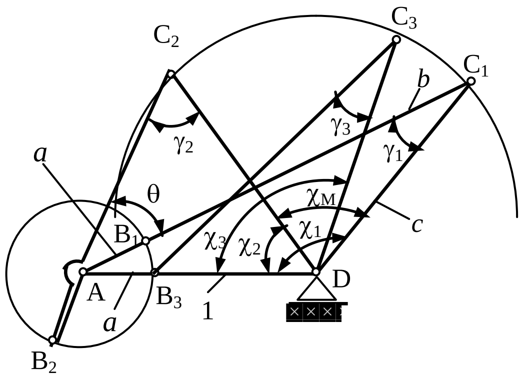

Fig. 7. The critical synthesis positions

We can also notice that the point B_3, where the mobile joint is located, coincides with the instantaneous rotation centre in the relative motion of the elements AB and CD.

Based on the fact that B_3 coincides with the instantaneous rotation centre, we can write the following relation of the absolute velocities in this point:

$$a\mathrm{d}\varphi = (1 - a)\,\mathrm{d}\chi \tag{24}$$

or

$$\frac{a}{1-a} = \frac{d\chi}{d\varphi} = i_3 \tag{25}$$

Hence, the relative length of the crank AB is:

$$a = \frac{i_3}{1+i_3} \tag{26}$$

The other sides of the mechanism, *b* and *c*, are determined from the triangle B_3C_3D, applying the sine theorem:

$$c = \frac{\sin(\chi_3 - \gamma_3)}{(1+i_3)\sin\gamma_3} \tag{27}$$

$$b = \frac{\sin\chi_3}{(1+i_3)\sin\gamma_3} \tag{28}$$

In the limit positions of the crank-rocker mechanism, AB_1C_1D and AB_2C_2D (Fig. 7), applying the sine theorem we can write the relations:

$$b + a = \frac{\sin \chi_1}{\sin \gamma_1} \tag{29}$$

$$b - a = \frac{\sin \chi_2}{\sin \gamma_2} \tag{30}$$

From relations (29) and (30), the following expressions result easily:

$$a = \frac{1}{2}\left[\frac{\sin \chi_1}{\sin \gamma_1} - \frac{\sin \chi_2}{\sin \gamma_2}\right] \tag{31}$$

$$b = \frac{1}{2}\left[\frac{\sin \chi_1}{\sin \gamma_1} + \frac{\sin \chi_2}{\sin \gamma_2}\right] \tag{32}$$

At the same times, applying the sine theorem in the same triangles, taking into accounts that:

$$\varphi = 180^0 - (\chi_1 + \gamma_1) \tag{33}$$

$$\varphi = 180^0 - (\chi_2 + \gamma_2) \tag{34}$$

we can write the following relations:

4. in the triangle AC_1D:

$$\frac{c}{\sin \varphi} = \frac{1}{\sin \gamma_1} \tag{35}$$

5. in the triangle AC_2D:

$$\frac{c}{\sin \varphi} = \frac{1}{\sin \gamma_2} \tag{36}$$

From relations (35) and (36), taking into account relations (33) and (34), it results:

$$c = \frac{\sin(\chi_1 + \gamma_1)}{\sin \gamma_1} = \frac{\sin(\gamma_2 + \chi_2)}{\sin \gamma_2} \tag{37}$$

From the three positions taken into account and considered critical positions AB_1C_1D, AB_2C_2D and AB_3C_3D, based on the formulas (26)÷(37), the following equalities can be written:

6. for the relative length *c*, from relations (27) and (37):

$$\frac{\sin(\chi_1+\gamma_1)}{\sin\gamma_1}=\frac{\sin(\chi_2+\gamma_2)}{\sin\gamma_2}=\frac{\sin(\chi_3+\gamma_3)}{(1+i_3)\sin\gamma_3} \tag{38}$$

7. for the relative length *b*, from relations (28) and (31):

$$\frac{1}{2}\left[\frac{\sin\chi_1}{\sin\gamma_1}+\frac{\sin\chi_2}{\sin\gamma_2}\right]=\frac{\sin\chi_3}{(1+i_3)\sin\gamma_3} \tag{39}$$

8. for the relative length *a*, from relations (26) and (32):

$$\frac{1}{2}\left[\frac{\sin\chi_1}{\sin\gamma_1}-\frac{\sin\chi_2}{\sin\gamma_2}\right]=\frac{i_3}{(1+i_3)} \tag{40}$$

Relations (38), (39) and (40) define the relative lengths *a*, *b* and *c* of the four bar crank-rocker mechanism. The rocker maximum oscillation angle $\chi_M = \chi_1 - \chi_2$, which is usually prescribed in the design requirements, can also be involved in these relations.

Thus, from the first equality (38), changing the mid-terms among themselves and implying the angle χ_M, we obtain:

$$\frac{\sin(\chi_1+\gamma_1)}{\sin(\chi_1-\chi_M+\gamma_2)}=\frac{\sin\gamma_1}{\sin\gamma_2} \tag{41}$$

Equality (40) can be written under the form:

$$\frac{1}{i_3}=\frac{2}{\dfrac{\sin\chi_1}{\sin\gamma_1}-\dfrac{\sin(\chi_1-\chi_M)}{\sin\gamma_2}}-1 \tag{42}$$

Equality (39), referring to the relative length *b*, can be arranged under the form:

$$\sin\chi_3=\frac{1+i_3}{2}\left[\frac{\sin\chi_1}{\sin\gamma_1}+\frac{\sin(\chi_1-\chi_M)}{\sin\gamma_2}\right]\sin\gamma_3 \tag{43}$$

Also, from the equality between the first and the third ratio of the relations (38), it results:

$$\cot\gamma_1=\frac{\sin(\chi_3-\gamma_3)}{(1+i_3)\sin\gamma_3\sin\chi_1}-\cot g\chi_1 \tag{44}$$

It is known that under design conditions, besides the minimum transmission angle $\gamma_{\min}$, the rocker maximum oscillation angle χ_M and the variation coefficient k of the average velocities at the advance and return stroke are imposed.

Writing down with θ the angle between the extreme positions B_1C_1 and B_2C_2 of the connecting rod BC, fig.7, corresponding to the extreme positions DC_1 and DC_2 of the rocker DC, the following two significant relations regarding the coefficient k can be written:

$$k = \frac{180^0 + \theta}{180 - \theta} \tag{45}$$

and

$$\theta = 180^0 \frac{k-1}{k+1} \tag{46}$$

From fig. 7 it is easy to see the following equality:

$$\chi_2 + \gamma_2 = \chi_1 + \gamma_1 - \theta \tag{47}$$

from which it results:

$$\gamma_2 = \chi_1 + \gamma_1 - \theta - (\chi_1 - \chi_M) = \chi_1 + \chi_M - \theta \tag{48}$$

Referring to the angle γ_3, it is known that it is the smallest transmission angle and consequently it should have the allowable value $\gamma_3 = \gamma_{ad}$.

The relations (41), (42), (43) and (44) make up a system of trigonometric equations with the unknowns χ_1, γ_1, χ_3 and i_3. Solving it can be done easily in the following sequence:

9. θ is calculated with the relation (46);
10. an arbitrary value of the angle $\gamma_1 > \gamma_{ad}$ is chosen;
11. γ_2 is calculated with the relation (48);
12. γ_1 is calculated with relation (41);
13. i_3 is calculated with relation (42);
14. χ_3 is calculated with relation (43);
15. γ_1, which we write down with γ_1^*, is calculated with relation (44).

In most cases, the value of the angle γ_1^* does not coincide with the initial chosen value γ_1. That is why a new value γ_1 will be chosen and the calculations will be repeated.

A new value γ_1^* will be obtained, which will be compared with the initial chosen value γ_1. The (increasing or decreasing) trends of the values γ_1^* will be seen in relation with the initially chosen value, repeating the calculations for other values till the condition $\gamma_1^* = \gamma_1$ is fulfilled.

3.1. Example of calculation

Determine the values of the relative lengths *a*, *b* and *c* of the sides of a four bar crank rocker to satisfy the following demands:

- the rocker oscillation angle, $\chi_M = 30^0$;
- the variation coefficient $k = 1.25$;
- the allowable minimum value of the transmission angle, $\gamma_{ad} = 45^0$.

3.1.1. Solution

1. $\gamma_1 = 70^0 > \gamma_{ad} = 45^0$ is chosen.
2. θ is calculated with relation (46): $\theta = 20^0$;
3. γ_2 is determined with relation (48): $\gamma_2 = 80^0$;
4. The rocker positioning angle χ_1 is calculated in the mechanism AB_1C_1D, limiting position with the following formula deduced from relation (41):

$$\chi_1 = arctg\left[\frac{\sin\gamma_1(\sin\gamma_2 + \sin(\chi_M + \gamma_2))}{\cos\gamma_1 \sin\gamma_2 - \sin\gamma_1 \cos(\chi_M + \gamma_2)}\right]$$

and then, with relation $\chi_1 - \chi_2 = \chi_M$, the rocker positioning angle χ_2 is calculated in the limiting position of the mechanism AB_2C_2D, $\chi_2 = \chi_1 - \chi_M$.

For the present case it results $\chi_1 = 37^0$ and $\chi_2 = 7^0$.

5. The parameters i_3, χ_3 and γ_1^* are calculated with formulas (42), (43) and (44), considering $\gamma_3 = \gamma_{ad}$.

For the present case it results: i_3=0.35; χ_3=22^0 and γ_1^*=76^0.

6. As the value found for γ_1^*=76^0 does not coincide with the chosen value γ_1=70^0 at point 1, the calculations should be repeated in order to find a new value γ_1.
7. In order to decrease the number of these repetitions, the graphics of the functions $\chi_1=\chi_1(\gamma_1)$ and $\gamma_1^*=\gamma_1^*(\chi_1)$ are plotted and their intersection point is determined. For the present problem we obtain the values: γ_1=60^0 and χ_1=53^0.
8. The geometric parameters searched *a*, *b* and *c* are calculated with formulas (26), (27) and (28).

For the present case: $a = 0.25$, $b = 0.66$, $c = 1.05$.

1. THE SYNTHESIS OF THE CRANK-SLIDER MECHANISM

In the case of the crank-slider mechanism as in the case of the four-bar mechanism it is required that the magnitude of the transmission angle should not be smaller than the allowable value. Besides, in most cases the mechanism should have a crank.

For the case where AB is the driving element, the mechanism transmission angle is the angle γ formed between the velocity direction of point C (v_C) and the direction of the relative velocity (v_{CB}).

From fig. 8 it results:

$$R \sin(\varphi_0+\varphi) + a = \sin(90^0-\gamma) \tag{49}$$

From relation (49), knowing that $-1 \le \sin(\varphi_0+\varphi) \le 1$, it results:

a) for $a > 0$:

$$\cos\gamma_{\min} = \frac{a+1}{l} \quad \text{and} \quad \cos\gamma_{\max} = \frac{a-1}{l} \tag{50}$$

b) for $a < 0$:

$$\cos\gamma_{\min} = \frac{a+1}{l} \quad \text{and} \quad \cos\gamma_{\max} = \frac{1-a}{l} \tag{51}$$

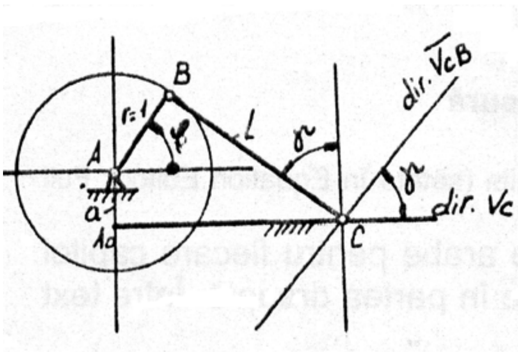

Fig. 8. Transmission angle

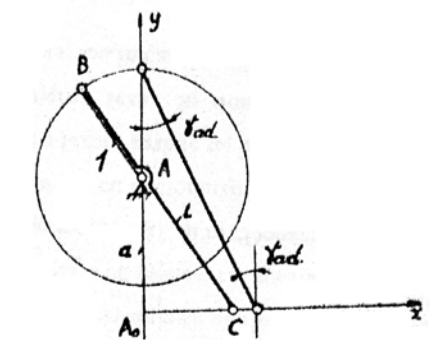

Fig. 9. The crank condition

Writing down the value of the allowable transmission angle with γ_{ad} and knowing that the sign of the cosine function does not count, we can write:

$$l \cos\gamma_{ad} \le a + 1 \tag{52}$$

In order to have a crank, the condition has to be fulfilled (fig. 9):

$$a < l\text{-}1 \tag{53}$$

If condition (53) is not fulfilled, the mechanism will be of the slider-rocker type.

From inequalities (52) and (53), it results that for the slider-crank mechanism to have crank and corresponding transmission angle the inequalities have to be met:

$$l \cos\gamma_{ad} \le a + 1 < l \tag{54}$$

The synthesis case depending on the average velocity variation coefficient and the transmission angle

In the following pages we present the algorithm for the synthesis of the crank-slider mechanism depending on the variation coefficient of the sliding block average velocities and the transmission angle magnitude.

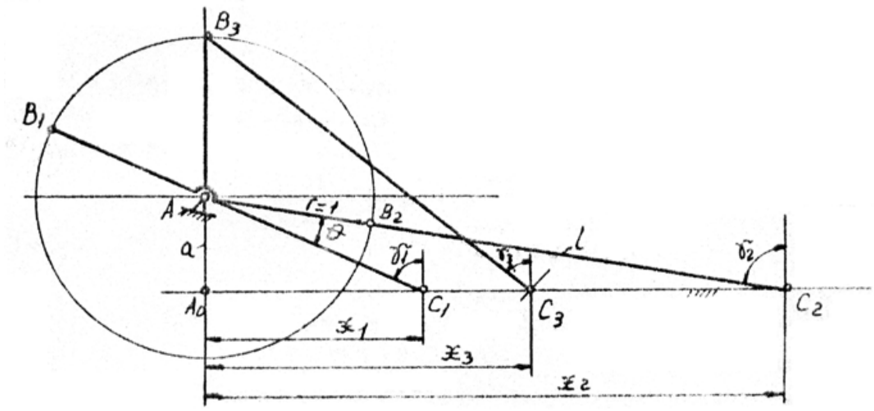

Fig. 10. Crank-slider mechanism

Suck problems arise with working mechanisms where both at the advance stroke and at the return stroke a force must be transmitted to the slider (for example, reciprocating pumps).

The problem is solved, taking into account three positions of the mechanism (fig.10): AB_1C_1, AB_2C_2, AB_3C_3.

We notice with θ the angle between extreme positions of the coupler and with γ the transmission angle. The distance A_0C is written down with X and it is considered that r = 1. From figure 10 it results:

$$a = X_1 \mathrm{ctg}\gamma_1 = X_2 \mathrm{ctg}\gamma_2 = X_3 \mathrm{ctg}\gamma_3 - 1 \tag{55}$$

and

$$l = \frac{X_1}{\sin\gamma_1} + 1 = \frac{X_2}{\sin\gamma_2} - 1 = \frac{X_3}{\sin\gamma_3} \tag{56}$$

Removing X_3 from (55) and (56) we get:

$$X_1 = \frac{1 - \cos\gamma_3}{\dfrac{\cos\gamma_3}{\sin\gamma_1} - ctg\gamma_1} \tag{57}$$

Removing X_2 from (56) and (55) and taking also into account that

$$\gamma_2 = \gamma_1 + \theta \tag{58}$$

after elementary transformations, we obtain:

$$X_1 = \frac{2}{\dfrac{\cot\gamma_1}{\cos(\gamma_1+\theta)} - \dfrac{1}{\sin\gamma_1}} \tag{59}$$

From relations (57) and (59) it is obtained:

$$\frac{\cos\gamma_1}{\cos(\gamma_1+\theta)} + 2\frac{\cos\gamma_1}{1-\cos\gamma_3} = 1 + 2\frac{\cos\gamma_3}{1-\cos\gamma_3} \tag{60}$$

As the synthesis conditions require that the minimum transmission angle (angle γ_3) should be equal with γ_{ad} and the angle θ is found from the variation coefficient of average velocities at the advance stroke and at the return stroke (known), relation (46), the value of the angle γ_1 can be found with relation (60).

Further on, the value of X_1 is found from equation (59) or equation (57). Then the unknowns *a* and *l* are found by means of the equations (55) and (56).

From equation (55), taking into account relation (58), we get:

$$X_2 = a \operatorname{tg}(\gamma_1+\theta) \tag{61}$$

The maximum slider displacement results:

$$H = X_2 - X_1 \tag{62}$$

4.1.1. Calculation example

Let us determine the *a* and *l* parameters of a crank-slider mechanism and the entire sliding block stroke for $k = 1.22$ and $\gamma_3 = \gamma_{ad} = 30^0$.

4.1.1.1. Solution

From relation (46) we get:

$$\theta = 180^0\,\frac{1.22-1}{1.22+1} = 17.8^0$$

From relation (60) it results $\gamma_1 \cong 70^0$. From relation (57) it results $X_1 = 0.33$. Then, from formulae (55) and (56) it results: $a = 0.09$ and $l = 1.26$. From relation (61) we get $X_2 = 2.2$. From relation (62) it results $H = 1.97$.

Calculation of the Mechanism Parameters when the Condition is Imposed that the Transmission Angle at the Advance Stroke Should Have a Value Equal to the Allowable Value

The advance stroke (fig. 11) begins in the position AB_1 and lasts up to position AB_2 in the direction of the arrow.

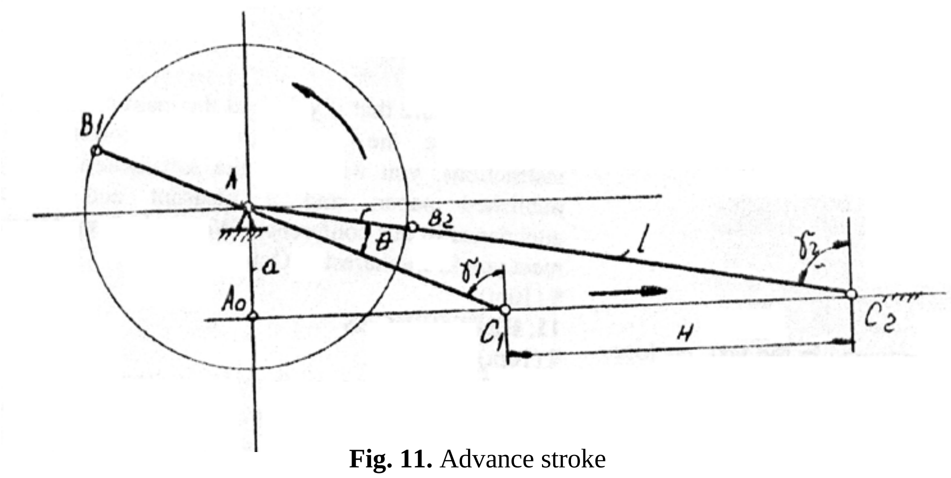

Fig. 11. Advance stroke

At the beginning of the stroke, the transmission angle is $\gamma = \gamma_1$. As it results from equation (49) on the interval $\varphi_1 = \varphi \leq 270^0$, the angle γ increases while on the interval $270^0 < \varphi < \varphi_2$ the angle γ decreases, reaching the value $\gamma_2 > \gamma_1$ at the end of the active stroke. Therefore, the lowest value of the angle γ is γ_1 and consequently angle $\gamma_1 = \gamma_{ad}$. Taking into account these remarks, the problem under consideration will be solved for two limit position: AB_1C_1 and AB_2C_2, taking $\gamma_1 = \gamma_{ad}$. To this end, the value X_1 is calculated with relation (60) and then the parameters *a* and *l* with relations (55) and (56).

We determine X_2 with formula (61) and then the sliding block stroke is found with relation (62).

4.2.1. Calculation Example

Let us determine *a* and *l* for $k = 1.4$ and $\gamma_1 = \gamma_{ad} = 45^0$ at the beginning of the active stroke.

4.2.1.1. Solution

With relations (46), (59), (55), (56), (61), (62) we find $\theta = 30^0$; $X_3 = 0.81$; $a = 0.81$; $l = 2.14$; $X_2 = 3$; $H = 2.19$.

Calculation of the mechanism parameters under the condition the transmission angle at the beginning of the working stroke should be equal to the allowable one

This case occurs when the working stroke is part of the sliding block advance stroke. The condition is set that during the working stroke the minimum transmission angle should not be smaller than the allowable angle.

From those shown above regarding the character of the transmission angle value variation during the advance stroke, it results that if the beginning of the working stroke coincides with the position AB_3C_3 (fig.12), then the angle γ_3 has to be equal with γ_{ad}.

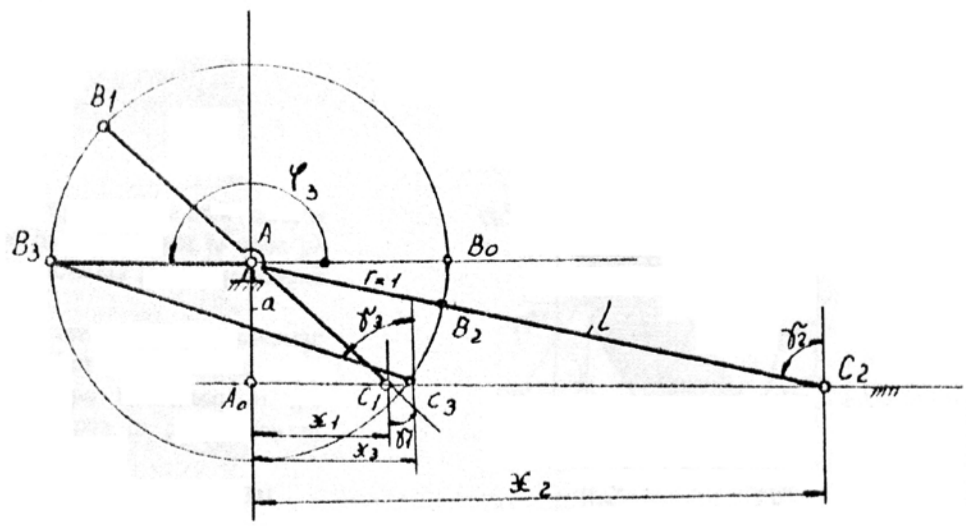

Fig. 12. The beginning of the working stroke

The transmission angles in all the other positions of the working stroke are greater than γ_{ad}. Therefore, the calculation positions in this case are: AB_1C_1; AB_2C_2; AB_3C_3. Besides, *k* and γ as well as the ratio between no-load run period at the advance stroke and the run period at advance (the corresponding sliding block displacement can also be considered) are known. This ratio is:

$$p = \frac{X_3 - X_1}{X_2 - X_1} \tag{63}$$

therefore

$$X_3 = (1 - p)\, X_1 = pX_2 \tag{64}$$

If the connecting rod BC in fig.13 is lengthened until it crosses A_0A in C_1, the triangle AC_1B can be considered to be the plane of mechanism velocities with the pole in A and rotated 90^0 counterclockwise.

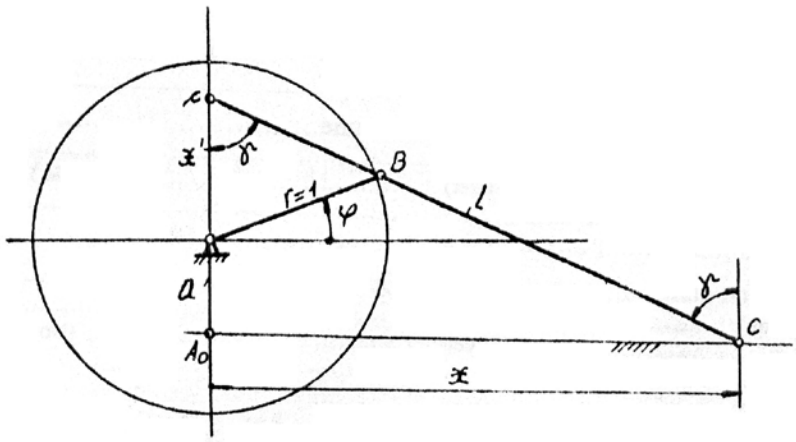

Fig. 13. The plane of mechanism velocities

Applying the sine theorem in this triangle, we obtain:

$$\frac{X^{'}}{\sin(90^0+\varphi-\gamma)}=\frac{1}{\sin\gamma},\quad \text{i.e.}\quad X^{'}=\frac{\cos(\varphi-\gamma)}{\sin\gamma} \tag{65}$$

From the triangle A_0C_1C it results:

$$a + X' = X \cot\gamma \ \text{ that is } a = X \cot\gamma - X' \tag{66}$$

By projecting the contour A_0ABC on the direction A_0C_1 we obtain:

$$l=\frac{X-\gamma\cos\varphi}{\sin\gamma} \tag{67}$$

Taking into account relations (64) and (65), relation (66) becomes:

$$a=\left[(1-p)X_1+pX_2\right]ctg\gamma_3-\frac{\cos(\varphi_3-\gamma_3)}{\sin\gamma_3} \tag{68}$$

In the same way, relation (67) becomes:

$$l=\frac{(1-p)X_1+pX_2-r\cos\varphi_3}{\sin\gamma_3} \tag{69}$$

But taking into account (58), from relation (56) we get:

$$l = \frac{X_2}{\sin(\gamma_1 + \theta)} - 1 = \frac{X_1}{\sin\gamma_1} + 1 \tag{70}$$

Taking into account relation (55), equation (61) can be written:

$$X_2 = X_1 \cot\gamma_1 \operatorname{tg}(\gamma_1+\theta) \tag{71}$$

By associating relations (55) and (68) we get:

$$X_1 ctg\gamma_1 = \left[(1-p)X_1 + pX_2\right]ctg\gamma_3 - \frac{\cos(\varphi_3 - \gamma_3)}{\sin\gamma_3} \tag{72}$$

By associating relations (69) and (70) it results:

$$\frac{X_1}{\sin\gamma_1} + 1 = \frac{(1-p)X_1 + pX_2 - \cos\varphi_3}{\sin\gamma_3} \tag{73}$$

By associating relations (71), (72), (73) and (59), a four equation system with four unknowns: X_1, X_2, γ_1 and φ_3 is obtained.

4.3.1. Sequence of calculation

The following steps are suggested:

1. A value is chosen for $\gamma_1 < \gamma_3 = \gamma_{ad}$.
2. X_1 and X_2 are found with relations (59) and (71).
3. From relations (72) and (73) we can write:

$$\cos(\varphi_3 - \gamma_3) = [(1-p)X_1 + pX_2]\cos\gamma_3 - X_1\cot\gamma_1\sin\gamma_3 \tag{73}$$

$$\cos\varphi_3 = (1-p)X_1 + pX_2 - \left(\frac{X_1}{\sin\gamma_1} + 1\right)\sin\gamma_3 \tag{74}$$

Equations (74) and (75) will have one and the same solution φ_3 only for a certain value of γ_1. This checking is also the appraisal criterion if the value chosen for γ_1 is the suitable one.

Let us suppose that after several tests a value has been established for γ_1 for which the two equations (74) and (75) have the same solution φ_3.

4. X_1 is found with relation (59) while X_2 is found with relation (71).
5. Stroke H is defined with relation (62).
6. The unknown parameters a and l are found with relations (55) and (56).

In case we cannot find the suitable value for γ_1 by the tests done so that equations (74) and (75) should yield one and the same value for φ_3, that is the synthesis conditions (in the present case values *k*, *p* and γ_3) cannot be met, the variation coefficient value of the average velocity *k* will be reduced or the value *p* will be modified.

5. DESIGN OF AN INVERTED SLIDE-CRANK MECHANISM: FOUR-BAR OSCILLATING-SLOTTED-LINK MECHANISM

5.1. Geometric synthesis

It is known that the process of fixing different links of a chain to create different mechanisms is called kinematics inversion.

The four-bar mechanisms are the four inversions of the four-bar chain and the slider-crank mechanisms are two inversions of the slider-crank mechanisms chain; three-bar and four-bar rotating when the lengths of the links comply with the condition $A_0A > A_0B_0$ respectively.

An inverted slider-crank mechanism is shown in figure 14. As the input crank A_0A makes a complete revolution, the crank B_0B', which is rigidly connected to the guide of the slider, oscillates through an angle. This output angle of oscillation can be determined by line B_0B_1, which is perpendicular to the guide. B_0B is called the eccentricity of the inverted slider-crank mechanism. The point B_1 is chosen for the suitable design of the „hardware", but point B is the point of significance. Figure 14 presents two extreme positions $A_0A_1B_1B_0$ and $A_0A_2B_2B_0$ of the inverted slider-crank mechanism, and points B_1 and B_2 are the extreme positions of the point B. The input and output crank angles between the two extreme positions are φ and χ, respectively. The transmission angle γ is also shown.

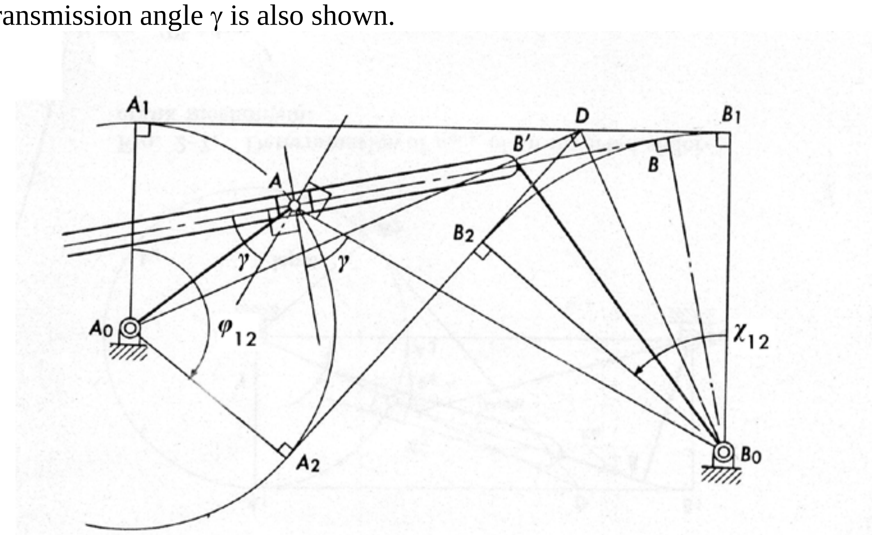

Fig. 14. Inverted slider-crank mechanism

Note $\langle A_0A_1B_1 = \langle A_1B_1B_0 = \langle A_0A_2B_2 = \langle A_2B_2B_0 = 90^0$.
Extend line A_2B_2 intersecting A_1B_1 at point D. Since

$$\langle A_1A_0A_2 + \langle A_1DA_2 = 180^0$$
$$\langle B_1B_0B_2 + \langle B_1DB_2 = 180^0$$
$$\langle B_1DB_2 + \langle A_1DA_2 = 180^0$$

then

$$\langle A_1DA_2 = \langle B_1B_0B_2 = \chi_{12} \tag{76}$$

$$\varphi_{12} + \chi_{12} = 180^0 \tag{77}$$

Again, since right triangle $B_0B_1D \equiv$ right triangle B_0B_2D and right triangle $A_0A_1D \equiv$ right triangle A_0A_2D, then

$$\langle A_1DA_0 = \langle A_2DA_0$$
$$\langle B_1DB_0 = \langle B_2DB_0$$

and therefore

$$\langle A_0DB_0 = 90^0 \tag{78}$$

The relationships (76), (77) and (78) establish the foundation for the design an inverted slider-crank mechanism to match prescribed dead-center positions. Two examples are used for illustration.

5.1.1. Example 1

Design an inverted slider-crank mechanism to meet the following requirements:

1. The length d of the fixed link A_0B_0 (fig.15).
2. Input crank length a.
3. The input crank angles φ_{12} between the two extreme positions.

5.1.1.1. Solution

1. At fixed pivot A_0, construct an isosceles triangle $A_1A_0A_2$, where $A_0A_1 = A_0A_2 = a$ and $\langle A_1A_0A_2 = \varphi_{12}$ (fig.15)
2. Erect lines A_1D and A_2D perpendicular to A_0A_1 and A_0A_2, respectively.
3. Join A_0D and erect DB_0 perpendicular to A_0D. DB_0 is the locus of the fixed pivot B_0 according to equation (78).
4. With A_0 as the center and length *d* as radius, strike a circular arc intersecting line DB_0 at B_0. Thus, B_0 is the fixed pivot.
5. The eccentricity *e* of the inverted slider-crank is the perpendicular distance from the fixed pivot B_0 to A_1D (fig.15).

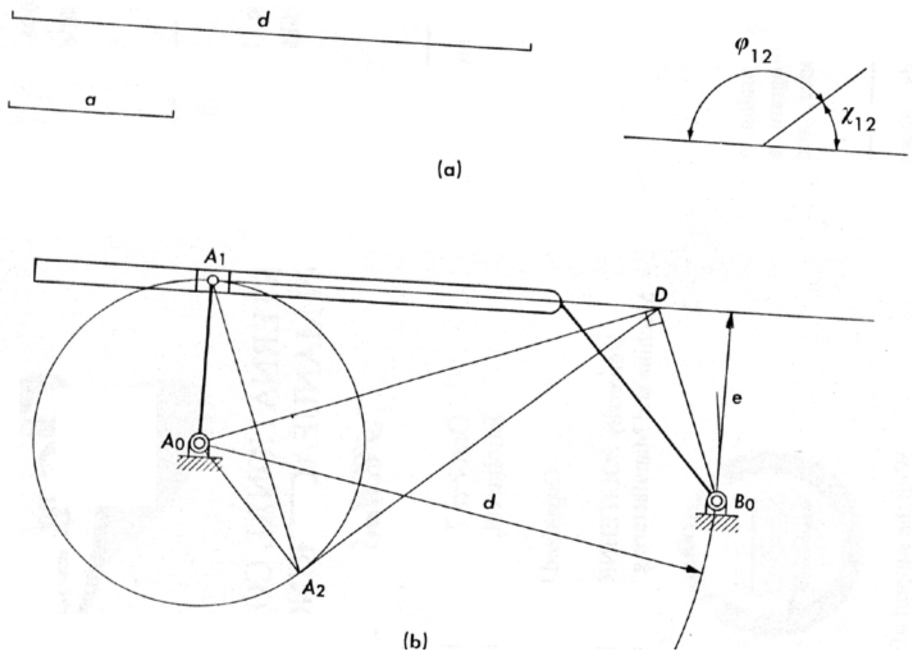

Fig. 15. Design of an inverted slider-crank mechanism

5.1.2. Example 2

Design an inverted slider-crank mechanism to meet the following requirements:

1. The eccentricity e of an inverted slider-crank mechanism (fig.16).
2. The output crank angle χ_{12} between the two extreme positions.
3. The crank length *a*.

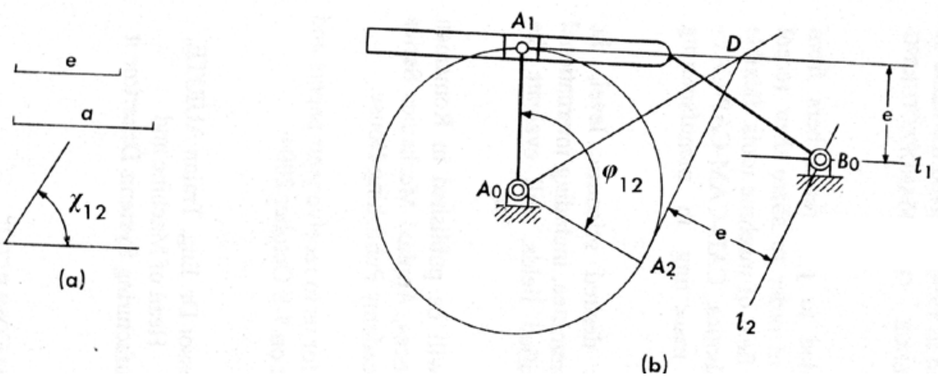

Fig. 16. Design of an inverted slider-crank mechanism

5.1.2.1. Solution

1. Since χ_{12} is given, φ_{12} can be found from the relationship of an inverted slider-crank mechanism: $\chi_{12} + \varphi_{12} = 180^0$. Thus, following the preceding design procedure, the point D can be located (fig.16).

2. Draw two lines l_1 and l_2 parallel to lines A_1D and A_2D at a distance *e*.

3. The intersecting point of lines l_1 and l_2 is the fixed pivot B_0 (fig.16).

5.2. Analytical synthesis

In fig.17 is shown an inverted slider-crank mechanism (four-bar-oscillating-slotted link mechanism) for analytic synthesis. With the notations in fig.17 we can write the vectorial equation:

$$\overline{BC} = \bar{e} + \bar{1} + \bar{a} \tag{80}$$

Squaring both members of the equation (80) and taking into account that:

$$BC^2 + a^2 = t^2 = a^2 + 1 - 2a\cos(\varphi_0+\varphi) \tag{81}$$

we obtain:

$$e - a\cos(\psi+\psi_0-\varphi_0-\varphi) + \cos(\psi+\psi_0) = 0 \tag{82}$$

From equation (82) it results that the four-bar-oscillating-slotted-link mechanism is dependent on four parameters, namely: *a*, *e*, φ_0 and ψ_0.

Applying the cosine theorem in the triangle ABD, the distance t is obtained:

$$t = \sqrt{a^2 + 1 - 2a\cos(\varphi + \varphi_0)} \tag{83}$$

In the case the element AB is the driving element, the transmission angle is the angle γ formed between the direction of the absolute velocity of the point B (perpendicular on t) and the direction of the relative velocity (parallel to the gliding direction of the gliding block BC). Rotating the sides of this angle clockwise, the angle formed between a perpendicular direction on the gliding direction BC and the direction BD is obtained. From fig.17 it results:

$$\cos\gamma = \frac{e}{t} \tag{84}$$

Taking into account relation (83), we shall obtain the expression of the transmission angle γ:

$$\cos\gamma = \frac{e}{\sqrt{a^2 + 1 - 2a\cos(\varphi + \varphi_0)}} \tag{85}$$

From relation (85), we infer that $\gamma = \gamma_{min}$ for $\varphi + \varphi_0 = 0$:

$$\cos\gamma_{\min} = \frac{e}{|a-1|} \tag{86}$$

Therefore, the condition of the transmission angle van is given by relation:

$$|a-1|\cos\gamma_{ad} \geq e \tag{87}$$

From fig.17 it results that mechanism has rotating crank lever if

$$a > 1 + e \tag{88}$$

From inequalities (87) and (88) we infer that for $a > 1$ and fulfilling condition (87), the four-bar rotating-slotted-link mechanism is obtained.

If

$$a < 1 - e \tag{89}$$

we shall have a four-bar oscillating-slotted link mechanism.

In order to fulfill the condition (87), it is necessary to fulfill the following inequality:

$$a \leq 1 - \frac{e}{\cos\gamma_{ad}} \tag{90}$$

which comprises also relation (89).

For the calculation of all the four parameters (a, e, φ_0 and ψ_0) of an inverted slider-crank mechanism, the position equation (82) is written under the form of the following interpolation polynomial:

$$p_0 + f_0(\varphi) + p_1 f_1(\varphi) + p_2 f_2(\varphi) + p_3 f_3(\varphi) = 0 \tag{91}$$

where

$$\begin{cases} p_0 = \dfrac{e}{\cos\psi_0} \\ p_1 = a(tg\psi_0 \cos\varphi_0 - \sin\varphi_0) \\ p_2 = a(tg\psi_0 \sin\varphi_0 + \cos\varphi_0) \\ p_3 = tg\psi_0 \end{cases} \qquad \begin{cases} f_0(\varphi) = \cos\psi \\ f_1(\varphi) = \sin(\psi - \varphi) \\ f_2(\varphi) = -\cos(\psi - \varphi) \\ f_3(\varphi) = -\sin\psi \end{cases} \tag{92}$$

Differentiating relation (91) successively in relation to φ, is obtained:

$$p_1 + \frac{1}{f_1'(\varphi)}\left[f_0'(\varphi) + p_2 f_2'(\varphi) + p_3 f_3'(\varphi)\right] = 0 \tag{93}$$

and

$$p_2 - \frac{1}{f_5(\varphi)}\left[f_4(\varphi) + p_3 f_3(\varphi)\right] = 0 \tag{94}$$

where

$$f_k(\varphi) = f_r''(\varphi) f_1'(\varphi) - f_r'(\varphi) f_1''(\varphi), \quad \begin{array}{c|c|c|c|} k & 4 & 5 & 6 \\ \hline r & 0 & 2 & 3 \end{array} \tag{95}$$

and

$$p_3 = \frac{f_4'(\varphi) f_5(\varphi) - f_4(\varphi) f_5'(\varphi)}{f_6'(\varphi) f_5(\varphi) - f_6(\varphi) f_5'(\varphi)} \tag{96}$$

where

$$f_k'(\varphi) = f_r'''(\varphi) f_1'(\varphi) - f_r'(\varphi) f_1'''(\varphi), \quad \begin{array}{c|c|c|c|} k & 4 & 5 & 6 \\ \hline r & 0 & 2 & 3 \end{array} \tag{97}$$

are the derivatives of the functions *f*(φ) (relations (92)).

The unknown parameters of the mechanism are determined with the formulae:

$$\begin{cases} tg\psi_0 = p_3 \\ e = p_0 \cos\psi_0 \\ tg(\varphi_0 - \psi_0) = \dfrac{p_1}{p_2} \\ a = \dfrac{p_2}{tg\varphi_0 \sin\varphi_0 + \cos\varphi_0} \end{cases} \tag{98}$$

6. SYNTHESIS OF THE SLOTTED-CRANK-TANGENT-GENERATOR MECHANISM

This mechanism is shown in figure 18. The driving element is the element AB. We can write with the notations in figure 18:

$$h + s - e\,\mathrm{tg}(\varphi_0 + \varphi) = 0 \tag{99}$$

The mechanism has three parameters: e, h and φ_0. For determining the three parameters, the equation (99) can be written under the form of the following interpolation polynom:

$$p_0 + f_0(\varphi) + p_1 f_1(\varphi) + p_2 f_2(\varphi) + p_3 f_3(\varphi) = 0 \quad (100)$$

where

$$\begin{cases} p_0 = etg\varphi_0 - h \\ p_1 = e + htg\varphi_0 \\ p_2 = tg\varphi_0 \end{cases} \quad \begin{cases} f_0(\varphi) = -s \\ f_1(\varphi) = tg\varphi \\ f_2(\varphi) = stg\varphi \end{cases} \quad (101)$$

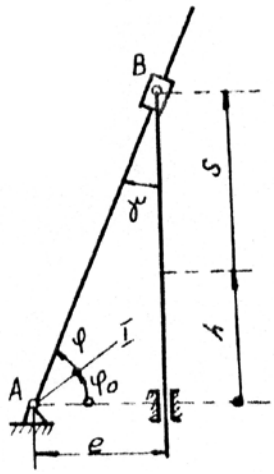

Fig. 18. Slotted-Link-Tangent Generator Mechanism

The coefficients p_0, p_1 and p_2 can be determined with the relations:

$$p_2 = -\frac{f_3(\varphi)}{f_4(\varphi)} \quad (102)$$

where

$$f_k(\varphi) = f_r''(\varphi) f_1'(\varphi) - f_r'(\varphi) f_1''(\varphi), \quad \frac{k \;|\; 3 \;|\; 4}{r \;|\; 0 \;|\; 2} \quad (103)$$

$$p_1 = -\frac{1}{f_1'(\varphi)}\left[f_0'(\varphi) + p_2 f_2'(\varphi)\right] \quad (104)$$

and p_0 is obtained from relation (100).

In these relations, the derivatives of the functions $f(\varphi)$ have the form:

$$\begin{cases} f_0'(\varphi) = s' \\ f_0''(\varphi) = s'' \\ f_1'(\varphi) = \dfrac{1}{\cos^2 \varphi} \\ f_1''(\varphi) = -2 f_1(\varphi) f_1'(\varphi) \\ f_2'(\varphi) = s' f_1(\varphi) - s f_1'(\varphi) \\ f_2''(\varphi) = s'' f_1'(\varphi) - s f_1''(\varphi) \end{cases} \quad (105)$$

The transmission angle results from relation:

$$ctg\gamma = \frac{h + s}{e} \quad (106)$$

$$a = \frac{1}{2}\left[\frac{\sin}{\sin}\right.$$

Therefore, in order to satisfy the condition of the minimum transmission angle, the relation must be fulfilled:

$$h + s_{\max} \leq e \cot\gamma_{ad} \tag{107}$$

where $s_{\max}$ is the maximum displacement a of the driven element, γ_{ad} the allowable transmission angle, h the distance from the reference angle to the initial position of the sliding block B.

7. CONCLUSIONS

From the previous discussion it is understood that $\gamma_{\min}$ should be equal to or larger than 40^0 or 45^0 in order to have effective force transmission on the working element. The examples shown above are simple ones.

For more complicated problems, such as linkage chains and adjustable linkages, the designer should sketch several possible types of mechanisms and then explore some approximate dimensional solutions for each mechanism. This process will soon rule out impractical mechanisms due to unfavorable minimum transmission angle. After he decides on one particular mechanism, several trials of the dimensions of certain links are usually necessary to yield a solution with the maximum $\gamma_{\min}$. This solution is the optimum design from the transmission angle standpoint.

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