THE HORIZON OF PREDICTION FOR EXCHANGE RATE EUR-LEU

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ABSTRACT: *For the chaotic systems, the fact that they are deterministic does not make them predictable. However, the predictive power in the case of chaotic systems can be improved and this can be illustrated by weather system for which predictions for short periods have reached to a very good accuracy. A positive Largest Lyapunov Exponent indicates the presence of chaos in the evolution of a time series and it is used to establish a correct horizon of prediction.*

KEY WORDS: *chaos; Lyapunov Exponent; exchange rate; horizon of prediction.*

JEL CLASSIFICATION: *C53; G17.*

1. INTRODUCTION

Chaotic systems are in fact complex deterministic systems with a large number of variables that influence the evolution of the process making it impossible for humans to simulate it and therefore making them to seem unpredictable. This, also, makes it impossible to determine the initial state of the system knowing just the final state.

Most processes and systems found in nature involve the interaction of many factors that allow us to catalog them as chaotic systems. Thus, chaos is met in solar system dynamics, evolution of populations, the weather, chemical reactions, etc.

In addition, the economy can be seen as a chaotic system, a factor that brings a huge number of variables is direct involvement of people. The chaos from complex systems is known as chaos deterministic.

For the chaotic systems, the fact that they are deterministic does not make them predictable. However, the predictive power in the case of chaotic systems can be improved and can be illustrate by weather system for which predictions for short periods have reached to a very good accuracy.

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We must emphasize the fact that the emergence and development of chaos theory could not have to take place before the invention of computers as simulation of complex systems with many variables could not have done without their help.

An important feature of chaotic systems is Sensitive Dependence on Initial Conditions (SDIC). This tells us that two initially close trajectories depart exponentially in a finite number of iterations, sometimes very quickly. In such a system, prediction is impossible except maybe the prediction for very short periods. The most used tool for identifying these processes from dynamical systems theory or experimental series is Lyapunov characteristic exponent (LCE).

Although chaos is fundamental deterministic, in reality is unpredictable except for short periods. Approximate time limit that can get accurate predictions for a chaotic system is a function of the largest Lyapunov exponent (Frison & Abarbanel, 1997)

$$\Delta t_{\max} = \frac{1}{\lambda_{\max}}.$$

2. MAXIMAL LYAPUNOV EXPONENT

Consider a model and two neighboring points $x_1(0),\ x_2(0)$ at the time $t=0$, starting points for two trajectories in phase space. Denote the distance between these two points *d(0)*. At the time *t*, that is after moving the two points along their trajectories, distance between points is measured again and denoted *d(t)*.

Using a different terminology, we can say that we applied a flow Φ_t to both points and after the time period *t* we measured the distance between the two points, *d(t)*.

The evolution of the relationship between the two distances is monitored.

$$\frac{d(0)}{d(t)} = e^{\chi t}$$

When *t* tends to infinity, χ converges to a value. The value of this limit is Lyapunov characteristic exponent.

If $\chi > 0$,it is said that the two orbits, initially close, diverge exponentially under the action of the flow. It also says that the Lyapunov characteristic exponent measures the rate of divergence of the system (Georgescu, 2012).

After calculating the Lyapunov maximum exponent or the determination of its approximations, we make assumptions about the nature of the system:

- $\lambda < 0$ The system generates a stable fixed point or a stable periodic orbit. Negative values of Lyapunov exponent are characteristic to non-conservative or dissipative systems. The higher the absolute value of the Lyapunov exponent the more stable is the system. A superstable fixed point will have a Lyapunov exponent that tends to minus infinity.
- $\lambda = 0$ A system with such an exponent is conservative.

- $\lambda > 0$ In this case, the orbits are unstable and chaotic. Points, initially very close, diverge to arbitrary values over time. A graphical representation is similar to a cloud of points without a distinct path.

2.1. Maximal Lyapunov Exponent for a differentiable function

Lyapunov exponent, $\lambda(x_0)$, measure the gap between the trajectories. Let x_0 and $x_0 + \varepsilon$ be two neighboring points. Lyapunov exponent satisfies the equality

$$\varepsilon e^{n\lambda(x_0)} = \left| f^n(x_0 + \varepsilon) - f^n(x_0) \right|.$$

Separating Lyapunov exponent and passing to the limit we obtain

$$\lambda(x_0) = \lim_{n\to\infty} \lim_{\varepsilon\to 0} \frac{1}{n} \ln \left| \frac{f^n(x_0 + \varepsilon) - f^n(x_0)}{\varepsilon} \right| = \lim_{n\to\infty} \frac{1}{n} \ln \left| \frac{df^n(x_0)}{dx} \right|.$$

Because $x_i = f^i(x_0)$ and $f^n(x_0) = f\left(f^{n-1}(x_0)\right)$, we have

$$\frac{df^n(x_0)}{dx} = f'(x_{n-1}) f'(x_{n-2}) \cdots f'(x_1) f'(x_0),$$

and for $\lambda(x_0)$ the calculation formula is written

$$\lambda(x_0) = \lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \ln \left| f'(x_i) \right|.$$

2.2. Maximal Lyapunov Exponent for a trajectory

For a trajectory $\{x_0, x_1, x_2, x_3, ...\}$ of points from $\mathbb{R}^d$ determined by an unknown

differentiable function $\Phi : \mathbb{R}^d \to \mathbb{R}^d$, can be estimated Lyapunov maximum exponent of

Φ at x_0 by iterative selection of non-negative integers $s(0), s(1), s(2), ...$ until the sequence of averages

$$\frac{1}{n}\sum_{t=0}^{n-1}\ln\frac{\left\|x_{s(t)+1}-x_{t+1}\right\|}{\left\|x_{s(t)}-x_t\right\|},$$

Show signs of convergence. The limit of this row would represent an estimate for the largest Lyapunov coefficient of the trajectory.

The row of averages estimate the global Lyapunov exponent of application Φ at x_0 relative to the direction $x_{s(0)}-x_0$, that is most likely the largest Lyapunov exponent for trajectory.

With the notation $x_{s(t)}-x_t = y_t$, we have

$$\frac{\left\|x_{s(t)+1}-x_{t+1}\right\|}{\left\|x_{s(t)}-x_t\right\|}=\frac{\left\|\Phi(x_t+y_t)-\Phi(x_t)\right\|}{\left\|y_t\right\|}.$$

The following present the way how can be chosen the values $s(t)$.

We consider two thresholds, one negative z^- and one positive z^+, by which we identify the distances too small and too large. It is chosen $s(0)$ so that the distance $\left\|x_{s(0)}-x_0\right\|$ to be as small as it can but greater than the minimum threshold z^-.

Is chosen $s(t)$ different from *t* and so that

$$x_{s(t)} = x_t + \delta_t\left(x_{s(t-1)+1}-x_t\right).$$

with δ_t small enough in absolute value to allow the estimation

$$\Phi\left(x_{s(t)}\right)-\Phi(x_t)\approx J(\Phi,x_t)\left(x_{s(t)}-x_t\right)$$

but large enough that the distance between $x_{s(t)}$ and x_t exceed the minimum threshold for noise removal, z^-.

It is considered the set $C_t=\left\{x_k \mid z^- < \left\|x_k - x_t\right\| < z^+\right\}$.

If $x_{s(t-1)+1}\in C_t$, $\delta_t = 1$ is small enough to satisfy the conditions and choose $s(t)=s(t-1)+1$. Otherwise, consider all the candidates from the set $\left\{x_{s(t-1)+1}\right\}\cup C_t$. From this set is chosen $s(t)$ so that $x_{s(t)}-x_t$ approximates a multiple of $x_{s(t-1)+1}-x_t$ which is small in absolute value.

All multiples of $x_{s(t-1)+1}-x_t$ are on a line passing through the origin. Also, all multiples of $x_k - x_t$ are on another line passing through the origin. Cosine of the angle between the two lines is

$$c_t(k) = \frac{\left(x_{s(t-1)+1} - x_t\right) \cdot \left(x_k - x_t\right)}{\left\|x_{s(t-1)+1} - x_t\right\| \left\|x_k - x_t\right\|}.$$

This quantity has values between -1 and 1. The choice of *k* for which $x_k - x_t$ is a good approximation of $x_{s(t-1)+1} - x_t$ must be such that the value of cosine to be as close as possible to 1.

Is ordered candidates for $s(t)$ using a partial order defined by the relation $\|x_i - x_t\| \le \|x_j - x_t\|$ and $c_t(i) \ge c_t(j) \Rightarrow i \succ j$. Is chosen $s(t)$ as the largest element according to this partial order.

2.3. Maximal Lyapunov Exponent for a time series

For a finite sequence $x_0, x_1, x_2, x_3, \ldots, x_N$ of points from $\mathbb{R}^d$ coming from an unknown differentiable function $\Phi : \mathbb{R}^d \to \mathbb{R}^d$, is proceeded as described previously only is no longer expected that the sequence of averages

$$\frac{1}{n} \sum_{t=0}^{n-1} \ln \frac{\left\|x_{s(t)+1} - x_{t+1}\right\|}{\left\|x_{s(t)} - x_t\right\|},$$

to show signs of convergence but simply the elements of this row are considered as approximations of the maximum Lyapunov exponent for the function Φ at x_0

3. MAXIMAL LYAPUNOV EXPONENT FOR THE TIME SERIES OF EXCHANGE RATE EUR-LEU

I used the time series of exchange rate EUR-LEU. Historical values of foreign exchange are available for download on the website of National Bank at http://www.bnr.ro/Baza-de-date-interactiva-604.aspx. For time series modeling and simulations I used MATLAB (R2011 a) and tstool Toolbox (time series tools).

The considered time series contains 3881 records during 04.01.1999 - 25.09.2012 and consists of exchange rate quotations of EUR-LEU established by National Bank of Romania for weekdays.

From the graphical representation (Figure 1.) we can see that in the case of the exchange rate EUR-LEU we deal with strongly nonlinear process. Nonlinearity does not necessarily imply chaos but any chaotic process is nonlinear.

The working mode for highlighting chaos in time series is sinking them into a multidimensional space.

Transition from one-dimensional time series to the corresponding *d*-dimensional series in state space is done using Takens theorem.

We embed one-dimensional series in a *d*-dimensional space by building vectors of length *d* as follows:

$$x_t^d = \left(x_t, x_{t+\tau}, \ldots, x_{t+\tau(d-1)}\right),\ t = 1,2,\ldots, N - \tau(d-1),$$

where τ is the number of time delays.

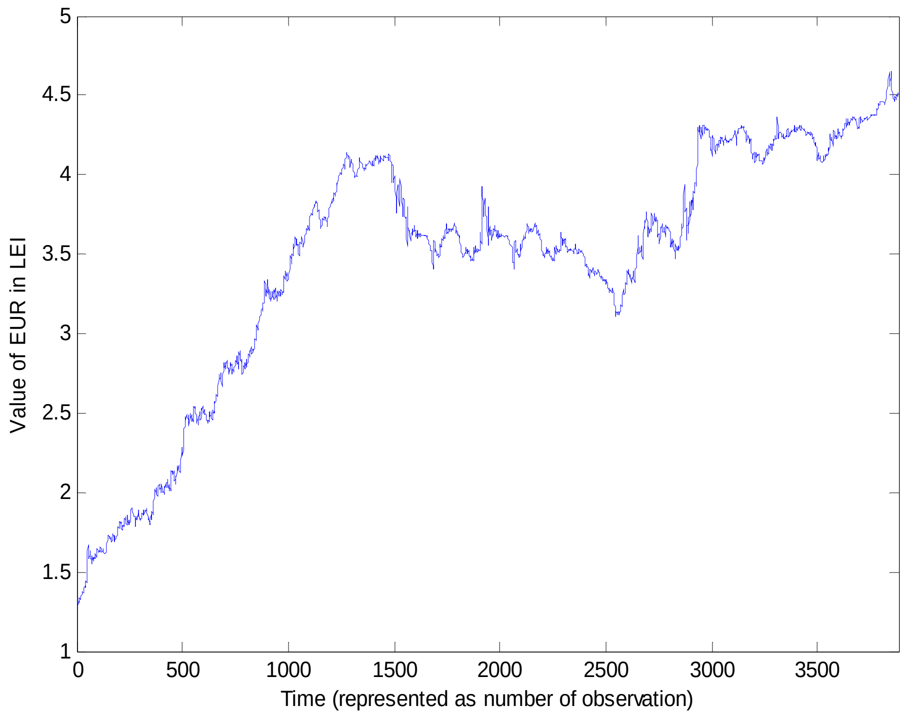

Figure 1. The evolution of exchange rate EUR-LEU over time

Identification of a suitable time delay is done by building auto-mutual information function and finding his first local minimum. I have conducted several simulations for different values of the embedding dimension and I have obtained values between 12 and 21. The most common value for the delay time was τ=19.

Using Cao’s method for determining the embedding dimension I obtained values between 5 and 7. I decided that minimum embedding dimension is 6, the value most often indicated by tests.

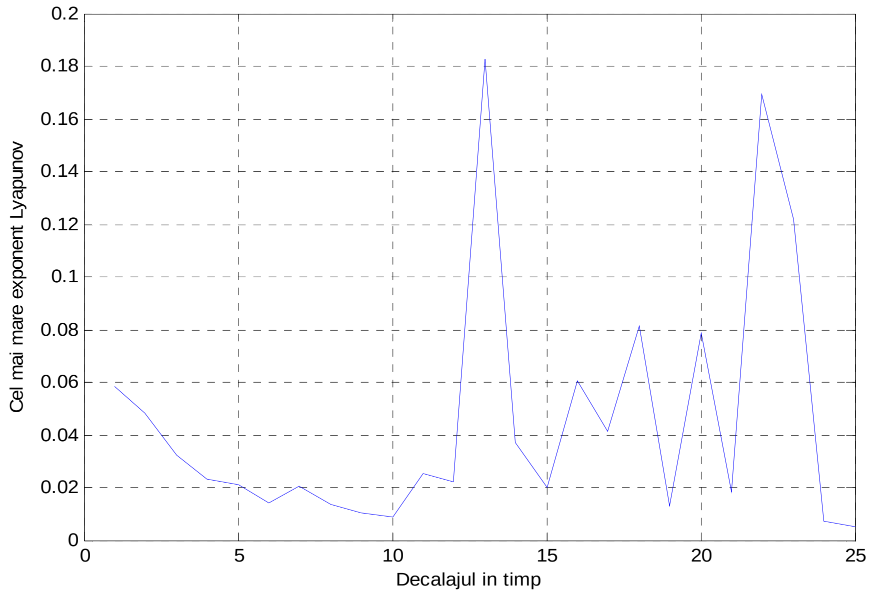

Figure 2. The Largest Lyapunov Exponent versus time delay

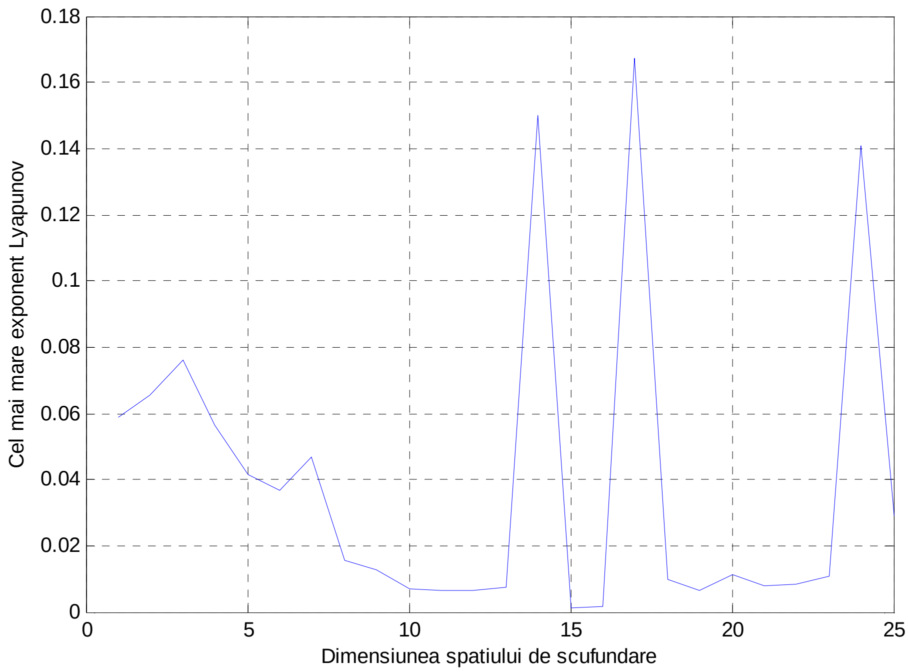

Figure 3. The Largest Lyapunov Exponent versus embedding space dimension

Figures 2 and 3 show the evolution of the largest Lyapunov exponent versus time delay respectively versus embedding space dimension. Positive values obtained are an indication of chaos for exchange rate time series of EUR-LEU.

4. CONCLUSIONS

In the case of the time series of the exchange rate between the euro and leu, simulations indicate the presence of chaos. With a maximum Lyapunov exponent of about 0.025, theoretically acceptable predictions are possible for a number of about 40 steps. Thus, remains open the problem of determining the model that simulates reasonably well the time series of the exchange rate, so that the predictions for first steps to be within acceptable error margin.

Determination of chaotic behavior is important to establish a correct prediction horizon

REFERENCES:

[1]. Abarbanel, H. (1996) *Analysis of observed chaotic data*, Springer, New York

[2]. Cao, L.; Mees, A.; Judd, K. (1997) *Modeling and Predicting Non-Stationary Time Series*, International Journal of Bifurcation and Chaos, Vol. 7, No. 8, pp. 1823-1831

[3]. Frison, T.; Abarbanel, H. (1997) *Ocean gravity waves: A nonlinear analysis of observations*, Journal of Geophysical Research, Vol. 102, No. C1, pp. 1051-1059

[4]. Georgescu, V. (2012) *Nonlinear Dynamics Chaos Theory Applications in Finance*, preprint

[5]. Goldsmith, M. (2009) *The Maximal Lyapunov Exponent of a Time Series*, A Thesis in The Department of Computer Science, Concordia University, Montreal, Canada

[6]. Lynch, S. (2004) *Dynamical systems with applications using MATLAB*, Birkhauser, 2004

[7]. Rosenstein, M.; Collins, J.; De Luca, C. (1992) *A practical method for calculating largest Lyapunov exponents from small data sets*, Phisica D, Vol. 65, pp. 117-134

[8]. Stuparu, D.; Vasile, T. (2009) *The Electronic Commerce in the Globalisation Era*, Annals of the University of Petroşani, Economics, 9(2), Universitas Publishing House, Petroşani, pp. 301-306

[9]. Turkes (Vint), M.C. (2010) *The Globalization of the Banking and Financial Crisis on International Level*, Annals of the University of Petroşani, Economics, 10(1), Universitas Publishing House, Petroşani, pp. 349-362

[10]. Vasilescu, M.; Mungiu-Pupăzan, M.C. (2010*) Inflation Targeting - Between Theory and Reality*, Annals of the University of Petroşani, Economics, 10(3), Universitas Publishing House, Petroşani, pp. 357-364