# BAYESIAN APPROACH OF DECISION PROBLEMS

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**ABSTRACT:** *Management is nowadays a basic vector of economic development, a concept frequently used in our country as well as all over the world. Indifferently of the hierarchical level at which the managerial process is manifested, decision represents its essential moment, the supreme act of managerial activity. Its can be met in all fields of activity, practically having an unlimited degree of coverage, and in all the functions of management. It is common knowledge that the activity of any type of manger, no matter the hierarchical level he occupies, represents a chain of interdependent decisions, their aim being the elimination or limitation of the influence of disturbing factors that may endanger the achievement of predetermined objectives, and the quality of managerial decisions condition the progress and viability of any enterprise. Therefore, one of the principal characteristics of a successful manager is his ability to adopt the most optimal decisions of high quality. The quality of managerial decisions are conditioned by the manager's general level of education and specialization, the manner in which they are preoccupied to assimilate the latest information and innovations in the domain of management's theory and practice and the applying of modern managerial methods and techniques in the activity of management. We are presenting below the analysis of decision problems in hazardous conditions in terms of Bayesian theory – a theory that uses the probabilistic calculus.*

**KEYWORDS:** *certitude; uncertainty; risk; decision; probability; Bayesian theory*

**JEL CLASSIFICATION:** *C11*

## 1. THE DECISION PROBLEM. THE CLASSIFICATION OF DECISIONS

By employing irreversibly material and human resources, with effects on the development of future activities, the *decision* could be defined like *a dynamic and deliberate thinking process that aims to correctly choose the optimal alternative from a*

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*number of options.* In other words, *decision is the social act through which groups of persons and material resources work together to realize a certain action* (Ceocea, 2010). Therefore, taking a decision implies the existence of at least two variants for achieving the objective.

In order to reduce at minimum perturbations, the decision also has to be made at levels as close as possible to the objective and in the shorter period of time, and also has to include the following logical elements: *abstraction or the rational reason* (formalization, modeling and simulation), *information* (acknowledging the reality as premise for choosing the optimal alternative) and *the objective* to achieve.

In this context, decisional factors that lead each subsystem, as well as the decision rules form the *management system*. No matter the nature of the assembly (economic, social etc.), it can not be considered automat systems (that can autocorrect itself) and, hence, the improvement of the performances is obtained through the permanent intervention of the management factors in the development of activities. Since the complexity and variety of interactions with the surrounding environment can not permit the aprioristic determination of all the rules for decision taking and its accomplishment points out the modification of some variables' values that are taking into account, it results the putting in practice of a decision is an actual *routing and control* function.

Uncertainty and insufficient knowledge of the possible consequences of all actions taken represent the principal difficulty in the process of elaborating and adopting a decision. Even in the conditions of a thorough analysis of all available information, there is always a certain degree of uncertainty that must be taken into consideration during the decision's elaboration process.

The determination of some mathematical models for the studying the phenomena with which the decisional factors from any field of activity are confronting in general, and especially those from the economic domain, is absolutely necessary, on one hand, for putting in an explicit and rigorous form the reports of causality between the factors that influence the consequences of a decision, and, on the other hand, for having available a scientific fundament for the decision, in this way being eliminated the routine, improvisation and subjectivism.

The decision problems can be differentiated from the others by a number of characteristics, which must be taken into consideration when a mathematical model has to be elaborated. A decision problem is considered to be defined formally, in general, if the following elements are clearly mentioned (Pekar&Smadici, 1995, pp.20-21):

**1. The Decision-maker (the decision factor)** who can be a person, a group of persons, a firm, an organization, etc.

**2. The decision-maker's possibilities of action**, by which can be understood different modalities through which the decision-maker can influence the behavior of the system towards which its actions are targeted. Each possibility of action is called an *alternative*, and the totality of the possibilities of action is known as *the lot of alternatives*. The act of choosing one of the alternatives, by which the decision-maker will act in the given situation, is called *decision*.

**3. Objectives.** The decision-maker's responsibility for the decision that is going to be taken can be defined like the necessity of choosing an alternative that will lead to obtaining some results; the evaluation of the decision's efficiency is based on those. The objectives targeted by the decision-maker consist, therefore, in emphasizing what kind of results he would like to obtain based on the decision he is going to take.

**4. The problem's context.** In concrete given situations, it rarely happens that the results of a decision to be exclusively determined by the decision taken, these results being influenced, more or less, by other elements (factors) that are under the control of the decision-maker only partially. These types of elements form what it can be considered the context of a problem, and we will call them *uncontrollable factors.* The economic decisions are usually taken depending on the reactions of some persons (consumers, competitors, salesmen) or on other uncontrollable factors, such as medium income, habits, traditions, trends, etc., which form together the context of the analyzed problem and it must be taken into consideration.

**5. Degree of uncertainty**. This aspect refers to the degree in which the behavior of the uncontrollable factors is unknown, and its effect on the consequences of decision. From this point of view, two categories of uncontrollable factors can be identified: uncontrollable factors for which the influence on the consequences of the decision are known in probabilistic sense and factors that have a completely unknown or insufficiently known behavior to which repartitions of probability that could characterize their influence on a decision's results can not be associated.

**6. Consequences' manner of evaluation.** Most of the times, the objectives targeted are presented through the description of the desired results' nature, frequently in a vague or ambiguous form: for example, it can be said that the growth of the turnover is targeted, but this will mean to explain what the turnover represents and how it is calculated. For the economic decisions, evaluation of possible results is done, usually, explicitly or implicitly, through the net benefits corresponding to those.

When the decision is taken, the decision-maker must take into account the context of the problem and, in this sense, an especially important role is hold by the *degree of uncertainty* on the consequences that might appear. The fact that most of the decisions relate to actions that will happen in the future leads to the manifestation of some unpredictable elements which determine the degree of uncertainty or the risk – on short, medium or long term. The degree of uncertainty ultimately reflects the decision-maker's level of knowledge regarding the relation between the possibilities of action at his disposition and the results (consequences) that can appear after choosing one of these possibilities of action. Taking into account the above mentioned, the decision problems are divided in three classes:

*i)* Decision problems *in conditions of certitude*, whenever the data of a problem are complete, have a high level of precision and lead to a single strategy for finding the optimum. A determinist economic-mathematical model will be used in this case;

*ii)* Decision problems *in conditions of uncertainty*, whenever there are relatively precise data that are still incomplete and the result of choosing an alternative can not be identify with precision. If the uncertainty can be measured from the probabilistic perspective, meaning the probability that a certain consequence will appear from a multitude of consequences after choosing an alternative can be determined, then it can be said this decision

problem is a *decision problem in hazardous/risky conditions*. A probabilistic mathematical problem will be used in this case.
*iii)* The decision problems in conditions of uncertainty, with a degree of uncertainty that can not be evaluated in probabilistic sense are called *decision problems in conditions of indeterminacy,* in which case a fuzzy (vague or informational) model will be used.

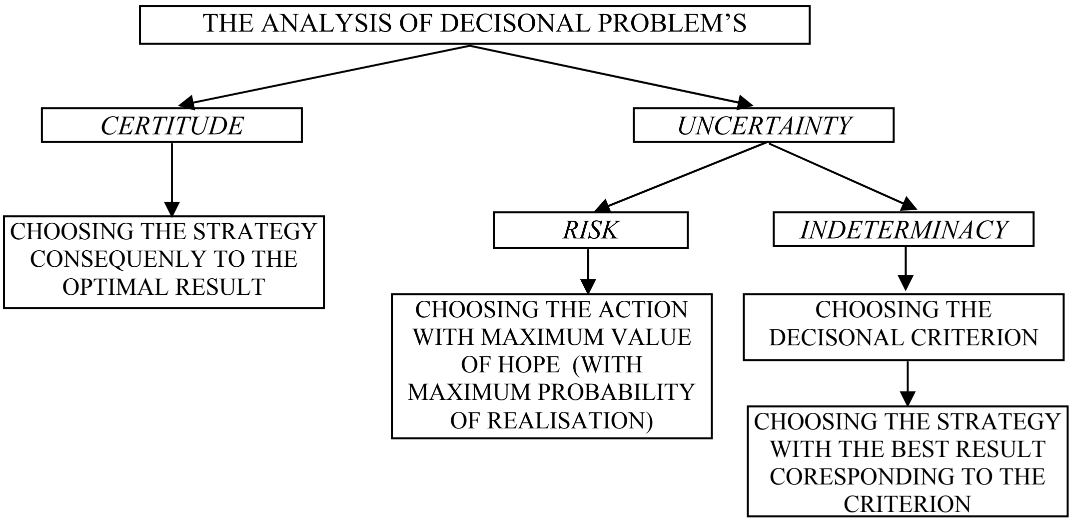


**Figure 1. Types of decisions**

The decisions in conditions of uncertainty are determined by the existence of a finite number of alternative strategies. Choosing the decision in conditions of uncertainty implies the prior assessment of the probability. The decisions in conditions of risk are usually referring to phenomena taking place on short terms (the frequency of spoilages, the dispersion of norms' realization etc.). The existence of information with low level of precision and still incomplete leads to the elaboration of heuristic decision models in which the intuition and experience of decision-maker have an important role. If precision and completeness of available data decrease simultaneously under a certain level, then either intuition or experience of the decision-maker can not capitalize on them; the decision will be taken at random.

## 2. ELABORATION STAGES OF THE DECISION

The process of taking a decision, pursuant to a defined objective, has clearly defined stages, as the following:

1. gathering the data;
2. selecting and ordering the data;
3. analyzing the data;
4. selection the solution and taking the decision;
5. putting the decision in practice and controlling the decision.

The first two stages define the analyzed problem, while the third assures the transformation of a particular situation in one with a general, and then individual, character. In the fourth stage, the optimization and adoption of the decision (transposing the real phenomenon real into a mathematic model, concomitantly with the reduction of the decision level) takes place, and in the fifth all action are identified.

The leader that has to take decisions must have the next indispensable aptitudes: the will to take a decision, the ability to calculate and the creativity. The more precise and clearly defined the problem is, the more reliable and accurate the decision becomes.

The analysis of the decision process's dominant characteristics conduct to divide the general methodology of the taking decision process in three stages: determinist, probabilistic and informational. Make these stages in this order ensure the taking of optimal decision. Make all three stages is not indispensable.

By monitoring the decision's analysis cycle, the following correlations could be made:

**Table 1. The three stages of the general methodology of the taking decision process**

| Determinist Stage | Probabilistic Stage | Informational Stage |
|---|---|---|
| The variables are well defined and correlated; the values are assigned; the importance of variables is evaluated without taking into consideration the uncertainty.<br>**Modeling**<br>Delimitation of the decision; nominalization of alternatives; determination of results; selection of variables; creation of structural models, preference of time values.<br>**Analysis**<br>Evaluation of the sensitivity to the decision variables and variables of state | Introduces probabilities for the important variables and deduce the estimations for the values' probabilities; introduces the preference of risk that guarantees the best solution in the stage of uncertainty.<br>**Modeling**<br>Coding the uncertainty in report to the variability of the data; coding the elements of risk<br>**Analysis**<br>Elaboration of the probabilistic distributions; evaluation of stochastic sensitivity; evaluation of the risk's sensitivity | Takes into consideration the results of the other stages to determine the economic value of uncertainty's elimination.<br>**Modeling**<br>Investigation of the modalities and possibilities of collecting the information.<br>**Analysis**<br>Evaluation of the economic sensitivity, respectively of the monetary value, so to be able to eliminate the uncertainty based on supplementary information |

## 3. THE BAYESIAN ANALYSIS OF A DECISION PROBLEM

### 3.1. Introduction

As far as the decision problems are concerned, we must also have a rational rule that will specify the manner in which an alternative must be chosen so to consider it the best (optimal) in the given situation. Such a rule is called *decision criterion*.

Most of the times, the economic decision problems are decision problems in conditions of uncertainty. The decision problems in conditions of uncertainty are also named "*games against nature*", by "nature" understanding the assembly of uncontrollable factors that can influence the result of the decision. It is obvious that, in order to take a correct decision, the decision taker should act in concordance with the laws governing the behavior of uncontrollable factors (nature) and to exploit these laws so to obtain the maximum benefit possible. However, in many situations, the decision-maker is not aware of these laws or has insufficient information relative to the manner in which those will react in the moment of putting into practice de decision. The fact the decision-maker does not have complete information relative to the consequences of choosing an alternative can be explained by the ignorance of the problem's context actual state. In the case of a concrete problem, it is taken into consideration a finite number of possible states of the problem's context $S_1$, $S_2$... $S_n$, these states being chosen so to be mutually exclusive and exhaustive, meaning the problem's context can be in only one of these states. If the decision-maker does not know the real state of the problem's context, the possible states $S_1$, $S_2$... $S_n$ could be identified with a complete system of events from a field of events. In other words, the states $S_1$, $S_2$... $S_n$ can be considered events incompatible two by two (they are mutually exclusive) and their reunion is a cert event (they are exhaustive).

Let's presume now the decision-maker would choose an alternative A and the state of the problem's context would be S, then a well determined consequence appears, and it is evaluated through a "benefit" (payment) noted with c(A, S). In the hypothesis the decision-maker has to choose between a finite number of alternatives $A_1$, $A_2$, ..., $A_m$, and for the problem's context a finite number of states $S_1$, $S_2$, ..., $S_n$ is taken into account, we will use notation $c_{ij}$ for the payments $c(A_i, S_j)$. So, $c_{ij}$ represents payment of the decision-maker when he chooses alternative $A_i$ and the problem's context actual state is $S_j$. Hence, the data of the problem can be synthesized in a matrix with m lines and n columns, each line corresponding to an alternative, and each column to a state of nature (context of the problems):

**Table 2. Payment Matrix**

| **Alternatives** | **States of nature** | | | |
|---|---|---|---|---|
| | $S_1$ | $S_2$ | | $S_n$ |
| $A_1$ | $c_{11}$ | $c_{12}$ | ... | $C_{1n}$ |
| $A_2$ | $c_{21}$ | $c_{22}$ | ... | $c_{2n}$ |
| ... | ... | ... | ... | ... |
| $A_m$ | $c_{m1}$ | $c_{m2}$ | ... | $c_{mn}$ |

The matrix $(c_{ij})_{m\times n}$ that includes all the possible payments of the decision-maker, depending on the alternative chosen and the actual state of nature, is also known as the *payment matrix* corresponding to the problem in discussion. The notion of payment matrix and not that of benefit matrix is used because there are situations in which the payment of the decision-maker expresses losses and not benefits (profits).

**Example 1:** Let's suppose that a firm F is preparing to launch on the market a new brand of product and it has to set the sale price. Because on the market there is already a brand of product with similar characteristics that is sold with the price *p*, it is considered that for the new brand it can be set a price bigger with 5 % than *p,* the same price *p*, or a price lower with 5% than *p*. The benefits of firm F will depend on the sale price of the new brand of product, but also on the reaction of the competitor, firm $F_C$, at the emergence on the market of this new trademark. We can presume the reaction of firm $F_C$ can be manifested by keeping the same price *p* for their own product, or by the diminishment of this price with 5% or 10%. In the next table are presented the hypothetical profits (the payment matrix) for firm F, in report with the price of the new trademark and the reaction of the competitor firm $F_C$.

**Table 3. The Payment Matrix for example 1**

| **Price Variations of the firm F** | **Price Variations of the competitor firm $F_C$** | | |
|---|---|---|---|
| | $S_1$: p | $S_2$: 0,95 p | $S_3$: 0,90 p |
| $A_1$: 1,05 p | 200 | 60 | 0 |
| $A_2$: p | 300 | 80 | 5 |
| $A_3$: 0,95 p | 250 | 100 | 10 |

If we don not have other information regarding the probable reaction of the competitor $F_C$ and we know only the data from the previous table, the problem of determining a rule based on which the firm F will choose one of the three variants of price emerges.

For the problems of marketing, it rarely happens the decision-maker has no type of information concerning the possible states of the problem's context. Even if this information are not sufficient to determine the objective estimations of the different states' probabilities, the decision-maker can use subjective estimation of those, the problem becoming therefore a decision problem in conditions of risk.

If the probabilities awarded to nature's states are determined only based on the information available before taking the decision, without taking into consideration the possibilities to obtain supplementary information, these probabilities are called *a priori probabilities* of nature's states. In the *Bayesian theory (theory that uses the calculus of probabilities)* of decision, the analysis of a decision problem in conditions of risk, based only on the information available before taking the decision, it is called *analysis a priori* of problem. When the probabilities of nature's states are determined based both on the a priori information, and the supplementary information that could be obtained relative to the problem's context, it is said an *analysis a posteriori* of problem.

### 3.2. The analysis a priori of a decision problem

For the decision problem in conditions of risk, the decision criterion usually used is criterion of the **maximum expected benefit** and, respectively, the criterion of the **minimum expected benefit**, when the elements of the payment matrix represent losses for the decision-maker.

In general, if the elements of the payment matrix represent benefits for the decision-maker, and the probabilities of states $S_1, S_2, ..., S_n$ in this matrix are noted by $p_1, p_2, \dots, p_n$, then the indicator of efficiency in the case of alternative $A_i$, noted $f(A_i)$ is considered to be the medium benefit expected, meaning $f(A_i) = \sum_j c_{ij} p_j$

Conform to the maximum expected benefit criterion, an alternative $A_{i0}$ is considered optimal if the highest medium expected benefit corresponds to it, meaning:

$$f(A_{i0}) = \max_i f(A_i) = \max_i \sum_j c_{ij} p_j$$

For example, let's suppose that, for the previous problem, the decision-maker (firm F) has information based on which it believes that the probabilities for the competitor firm $F_C$ to choose one of the prices corresponding to states $S_1, S_2, S_3$ are the following:

$$p(S_1)=p_1=0{,}6 \qquad p(S_2)=p_2=0{,}3 \qquad p(S_3)=p_3=0{,}1 \qquad (1)$$

The expected benefits associated to each of the three alternatives are calculated as below:

$$\begin{aligned} f(A_1) &= 200\times0{,}6+60\times0{,}3+0\times0{,}1=138 \\ f(A_2) &= 300\times0{,}6+80\times0{,}3+5\times0{,}1=204{,}5 \\ f(A_3) &= 250\times0{,}6+100\times0{,}3+10\times0{,}1=181 \end{aligned} \qquad (2)$$

Applying for this problem the criterion of the maximum expected benefit, we identify the optimal alternative as $A_2$ (determining a price equal with the present price of competitor firm).

It was considered, in the conditions of the given problem, that the probabilities for the competitor firm to choose one of the three prices, correspondent to states $S_1$, $S_2$, $S_3$, do not depend on the variant of price the firm F would chose and this fact is unrealistic. It is obvious the firm $F_C$ will react to the challenge of firm F and the price adopted by it will also depend on the firm F's variant of price. In this situation, instead of the probabilities $P(S_j)$ we must take into account the conditional probabilities $P(S_j/A_i)$ (probability that firm $F_C$ adopts the price corresponding to state $S_j$, in the hypothesis the firm F chooses the variant of price corresponding to alternative $A_i$). We can assume, for example, that those conditional probabilities are given in table 4. Because states $S_1$, $S_2$, $S_3$ can be considered as a complete system of events, the sum of the conditional probabilities from each line must be equal to the unit.

**Table 4. The conditional probabilities for example 1**

| **Price variations of firm F** | **Price Variations of competitor firm $F_C$** | | | |
|---|---|---|---|---|
| | $S_1$: p | $S_2$: 0,95 p | $S_3$: 0,90 p | Total |
| $A_1$: 1,05 p | 0,7 | 0,2 | 0,1 | 1 |
| $A_2$: p | 0,4 | 0,2 | 0,4 | 1 |
| $A_3$: 0,95 p | 0,2 | 0,3 | 0,5 | 1 |

By using the expected benefits matrix from table 3 and the conditional probabilities from table 4, the benefits corresponding to each alternative become:

$$f(A_1)= 200\times0{,}7+60\times0{,}2+0\times0{,}1=152$$
$$f(A_2)=300\times0{,}4+80\times0{,}2+5\times0{,}4=138 \quad (3)$$
$$f(A_3)=250\times0{,}2+100\times0{,}3+10\times0{,}5=85$$

In this situation, the optimal alternative is $A_1$, because the highest expected benefit corresponds to it.

### 3.3. The analysis a posteriori of a decision problem

We should suppose that, for a decision problem in conditions of risk, the decision-maker has considered he needs supplementary information and he is in their possession. The analysis a posteriori is referring to the manner in which the decision-maker can use this information to correct the probabilities a priori of the problem's context possible states. The probabilities that are corrected, also known as probabilities a posteriori, are going to be used for choosing the optimal alternative. In order to determine the probabilities a posteriori, it is used the Bayes formula from the probabilities theory, on which is based the Bayesian theory of decision.

Let S be one of the possible states of nature and note the probability a priori of this state with P(S). We also should assume now the decision-maker has decided to obtain supplementary information, which lead to the formulation of a conclusion, noted with X, from the total lot of possible conclusions (results). Taking to account this result, the correction of the state S's probability a priori can be made by replacing it with the conditioned probability P(S/X), this being named probability a posteriori of state S. On the other hand, by using the properties of the conditional probabilities, we can write:

$$P(S/X)=\frac{P(S)\cdot P(X/S)}{P(X)} \quad (4)$$

And it would be sufficient, therefore, to know the probabilities P(X/S) and P(X), where P(X/S) is the probability to obtain result X in the hypothesis the real state of nature is S, so to determine the probability a posteriori of state S. Let us assume now there are a finite number of possible states of nature $S_1, S_2, ..., S_n$ with the probabilities a priori $P(S_1), P(S_2), ..., P(S_n)$. If the conditional probabilities $P(X/S_i)$, i=1, 2, ..., n would be known, then the probability the supplementary information to lead to result X can be determined by using the formula of total probability:

$$P(X) = P(S_1)\, P(X/S_1) +P(S_2)\, P(X/S_2) + ... + P(S_n)\, P(X/S_n) \quad (5)$$

Hence, if the probabilities a priori of states $S_1, S_2, ..., S_n$ and the conditional probabilities $P(X/S_i)$, i=1, 2, ..., n, are known, then the probabilities a posteriori of the nature's state can be calculated by using the Bayes formula:

$$P(S_i / X) = \frac{P(S_i) \cdot P(X / S_i)}{P(X)}, \ i=1, 2, ..., n \tag{6}$$

where P(X) is calculated with the formula above (5).

**Example 2:** Let's suppose a firm is going to choose between three variants for the determination of the capacity of production necessary for a new brand (product), which will be noted $A_1$, $A_2$, $A_3$. The profitability of the new product depends on the capacity of production that is to be determined, but also on the level of the demand for the respective product, which is still unknown, but it is considered that knowing the demand is at high, medium or low level would be enough. We will note with $S_1$, $S_2$, $S_3$ the three possible states of demand, corresponding to the demands at high, medium or low level. Depending on the alternative chosen for the capacity of production and the problem's context possible states, it is assumed the benefits that would be made by the firm are represented by the data from table 5. Based on the existing information regarding the consumers' behavior towards the introduction on the market of a similar brand, it is also assumed the probabilities a priori for the real demand are those from the last line of the table:

**Table 5. The Payment Matrix for example 2**

| Alternative | States of the problem's context | | | Expected Benefit |
|---|---|---|---|---|
| | $S_1$ | $S_2$ | $S_3$ | |
| A1 | 600 | 250 | -100 | 250 |
| A2 | 400 | 250 | 0 | 200 |
| A3 | 100 | 80 | 50 | 80 |
| $P(S_i)$ | 0,3 | 0,5 | 0,2 | |

It can be observed from the last column of table 5 that, consequently to the analysis a priori, it is recommended for the decision-maker to choose alternative $A_1$ (high level capacity), since the maximum expected benefit corresponds to it. However, this alternative is the only one for which there are 20% chances that, instead of benefits, the firm will record losses, and due to this fact, it has been decided that some supplementary investigations about the demand's real level must be made. The data obtained could not lead to the exact determination of the demand's level, but an evaluation of the precision with which these data can be used to estimate the level of demand was done. For example, the investigations lead to the fact that in the hypothesis the real level of demand is high, the data confirms a high level in 90% of the cases.

To express in a more concise form the possible situations, we are going to set some notations. We note with $X_1$, $X_2$, $X_3$ the events that are taking place when the data obtained as consequence of the research made, confirm a high, medium and low level of demand. The fact that, in the hypothesis the real state of the demand is $S_1$ (high level demand), this state is confirmed in 90% of the cases and that, in 10% of the cases, the data can lead to the conclusion demand would be at medium level can be expressed by using the usual notations for conditioned probabilities:

$$P(X_1/S_1)=0{,}90;\ P(X_2/S_1)=0{,}10 \tag{7}$$

On the other hand, if the demand's real level is high, the data gathered can not lead to the conclusion that it would be at low level, meaning we can write $P(X_3/S_1)=0$. In table 6 are given the values of all conditional probabilities $P(X_j/S_i)$:

**Table 6. The conditional probabilities for example 2**

| The real level of demand | $P(X_1/S_i)$ | $P(X_2/S_i)$ | $P(X_3/S_i)$ | Probabilities a priori of the states $P(S_i)$ |
|---|---|---|---|---|
| $S_1$ | 0,90 | 0,10 | 0 | 0,3 |
| $S_2$ | 0,15 | 0,80 | 0,05 | 0,5 |
| $S_3$ | 0,05 | 0,20 | 0,75 | 0,2 |

Let's assume now that, consequently to the research made, the conclusion the demand is at medium level was reached, meaning the result $X_2$ was obtained. The probabilities a posteriori of states $S_1$, $S_2$, $S_3$ will be then:

$$P(S_i / X_2) = \frac{P(S_i)P(X_2 / S_i)}{P(X_2)}, \quad i=1, 2, 3 \tag{8}$$

where:

$$P(X_2)=P(S_1)P(X_2/S_1)+P(S_2)P(X_2/S_2)+ P(S_3)P(X_2/S_3)=0{,}3\cdot 0{,}1+0{,}5\cdot 0{,}8+0{,}2\cdot 0{,}2=0{,}47 \tag{9}$$

So:

$$\begin{aligned} P(S_1 / X_2) &= \frac{0{,}3 \cdot 0{,}1}{0{,}47} = 0{,}064 \\ P(S_2 / X_2) &= \frac{0{,}5 \cdot 0{,}8}{0{,}47} = 0{,}851 \\ P(S_3 / X_2) &= \frac{0{,}2 \cdot 0{,}2}{0{,}47} = 0{,}085 \end{aligned} \tag{10}$$

The probabilities a posteriori being determined, we can go further on to the determination a posteriori of the expected benefits corresponding to each alternative $A_i$, i=1, 2, 3:

$$\begin{aligned} f(A_1) &= P(S_1/X_2)\cdot S_1+ P(S_2/X_2)\cdot S_2+ P(S_3/X_2)\cdot S_3= \\ &= 0{,}064\cdot 600+0{,}851\cdot 250+0{,}085\cdot(-100)=251{,}15 \\ f(A_2) &= P(S_1/X_2)\cdot S_1+ P(S_2/X_2)\cdot S_2+ P(S_3/X_2)\cdot S_3= \\ &= 0{,}064\cdot 400+0{,}851\cdot 250+0{,}085\cdot 0=238{,}35 \\ f(A_3) &= P(S_1/X_2)\cdot S_1+ P(S_2/X_2)\cdot S_2+ P(S_3/X_2)\cdot S_3= \\ &=0{,}064\cdot 100+0{,}851\cdot 80+0{,}085\cdot 50=78{,}73 \end{aligned} \tag{11}$$

The final results are exposed in table 7:

**Table 7. Expected benefit after a posteriori analysis for example 2**

| **Alternative** | **The states of problem's context** $S_1$ | $S_2$ | $S_3$ | **Expected benefit a posteriori $f(A_i)$** |
|---|---|---|---|---|
| A1 | 600 | 250 | -100 | 251,15 |
| A2 | 400 | 250 | 0 | 238,35 |
| A3 | 100 | 80 | 50 | 78,73 |
| Probabilities a posteriori $P(S_i/X_2)$ | 0,064 | 0,851 | 0,085 | |

The analysis a posteriori leads us to the conclusion the optimal alternative is $A_1$, but, differently from the analysis a priori, we discover that the risk to lose 100 monetary units was reduced from 20% to 8,5% and this fact can strengthen the decision-maker's belief he has made the right choice.

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